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Game Theory with gradient based optimization algorithm

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OUTLINE

- INTRODUCTION
- THEORY
  - *NASH THEORY*
  - *SIMPLE GRADIENT METHOD*
  - *DACE MODEL*
- TEST
- APPLICATION
- CONCLUSION

INTRODUCTION

REDUCTION OF NUMBER OF DESIGNS
TO COMPUTE FOR OBTAINING GOOD RESULTS
BY OPTIMIZATION

MULTI OBJECTIVE OPTIMIZATION

INTEGRATION OF CAD, CAE, CFD, etc. TECHNIQUES
WITH NUMERIC OPTIMIZATION METHODS WITH
ADVANTAGE FOR INDUSTRY

INTRODUCTION

NASH THEORY

multi objective optimization

mono objective optimization

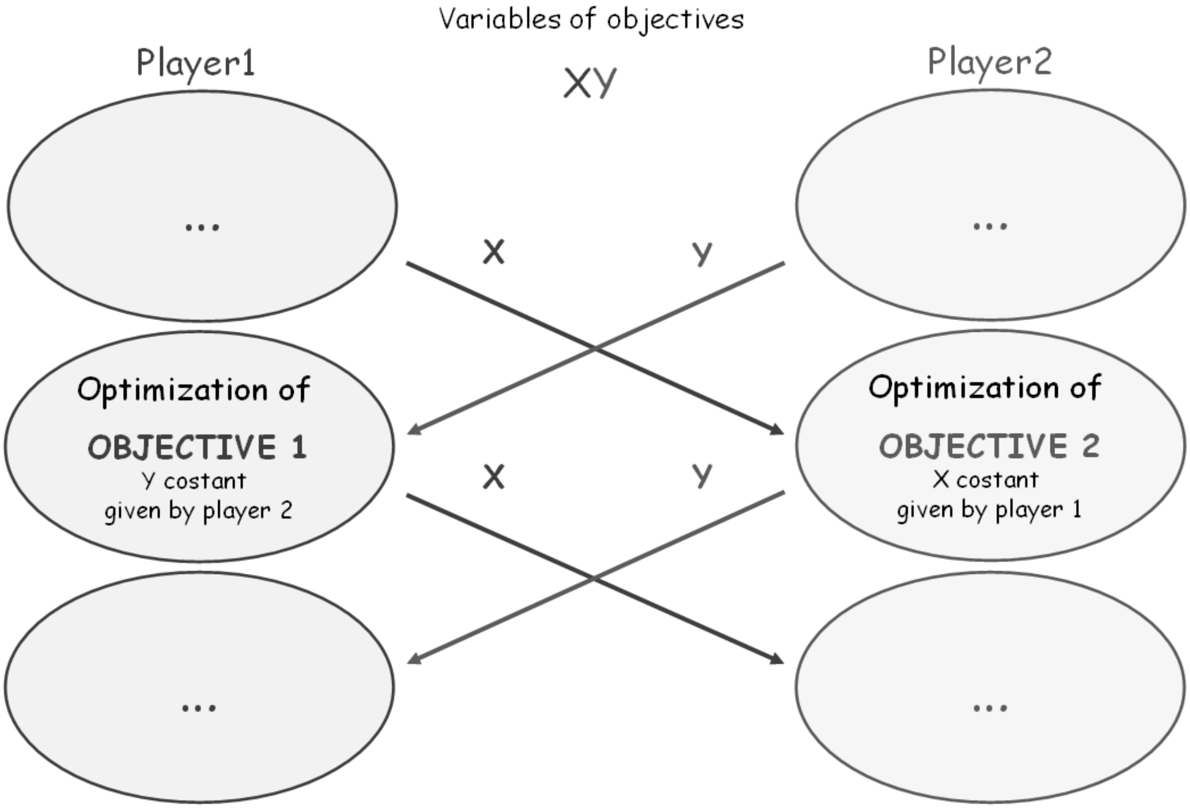
GRADIENT METHOD

RESPONSE SURFACES

GAME THEORY

NASH THEORY

non-cooperative   symmetric



NASH THEORY

Consider two-player Nash game

$$f_A(x, y) : AxB \Rightarrow \Re$$
$$f_B(x, y) : AxB \Rightarrow \Re$$

are the objective functions to minimize

A search space for first player
B search space for second player

$(x_*, y_*) \in AxB$ is NASH EQUILIBRIUM if:

$$f_A(x_*, y_*) = \inf_{x \in A} f_A(x, y_*)$$
$$f_B(x_*, y_*) = \inf_{y \in B} f_B(x_*, y)$$

where f_A gain for first player

f_B gain for second player

GRADIENT METHOD

iterative method :

$$X^{(K+1)} = X^{(K)} + a^{(K)} S(X^{(K)})$$

- $X^{(K)}$ actual variables
- $X^{(K+1)}$ estimated variables for next step
- $a^{(K)}$ step length
- $S(X)$ searching function of right direction

GRADIENT METHOD

SIMPLE GRADIENT METHOD

$$X^{(K+1)} = X^{(K)} - a^{(K)} \nabla f(X^{(K)})$$

where α is costant and gradient is the searching function

computing of partial derivatives by

central finite difference method

on DACE model

COMPUTATIONAL SAVING

DACE MODEL

modelling phenomena by

SRF Spatial Random Fields

simplifing hypoteses

linear estimators

$$Y(x)=\sum_{h=1}^{k} b_h f_h(x)+e(x)$$

CORRELATION FUNCTION between errors:

$$Corr[e(x(i)),e(x(j))]=\exp[-d(x(i),x(j))]$$

$$d(x(i),x(j))=\sum_{h=1}^{k} q_h |x_h(i)-x_h(j)|^{p_h} \qquad q_h>0 \quad p_h\in[1,2]$$

LIKELYHOOD:

$$\frac{1}{(2p)^{n/2} s^{2^{n/2}} (\det R)^{1/2}} \exp \frac{(y-1m)' R^{-1}(y-1m)}{s^{2}}$$

MAX → θ,p

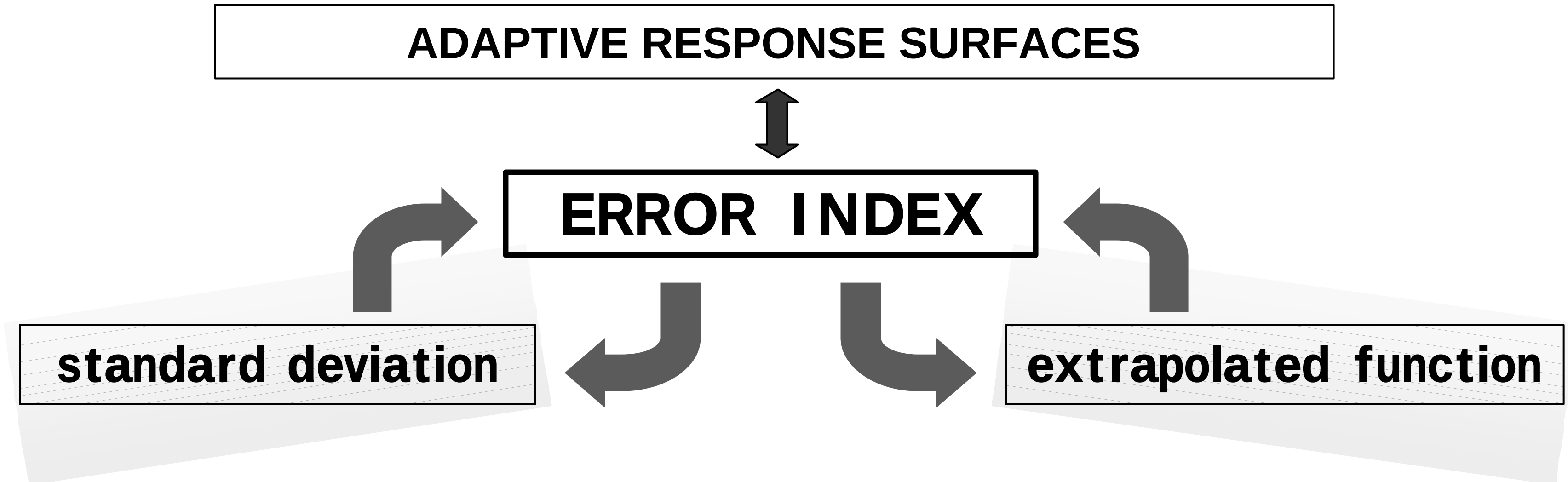
BEST LINEAR UNBIASED PREDICTOR

$$\hat{y}(x^*)=\hat{\mu}+r' R^{-1}(y-1\hat{\mu})$$

MEAN SQUARED ERROR OF THE PREDICTOR

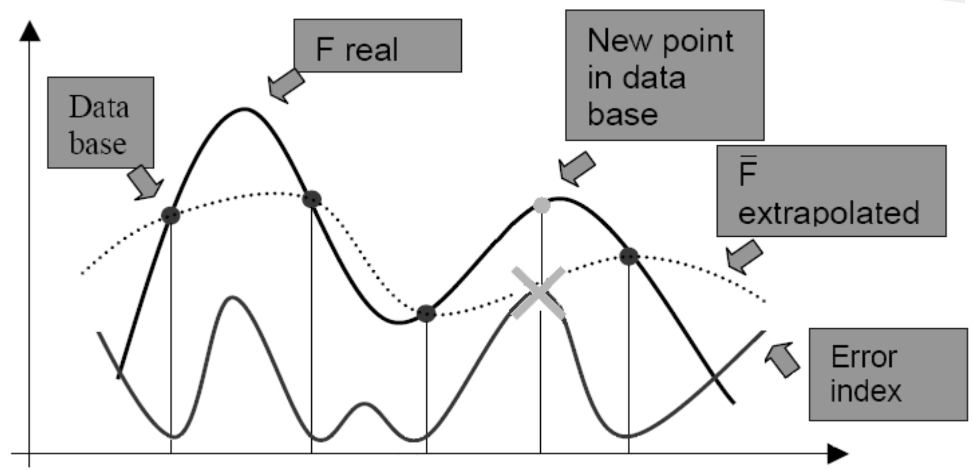
$$s^2(x^*)=\sigma^2\left[1-r' R^{-1} r+\frac{(1-1R^{-1}r)^2}{1' R^{-1} 1}\right.$$

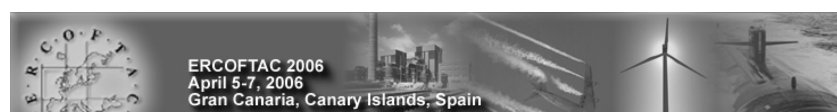
ADAPTIVE DACE MODEL



If f is to maximize:

$$IE = \frac{f_{\max} - f}{f_{\max} - f_{\min}} \cdot \frac{s}{s_{\max}}$$





ALGORITHM

Resuming..

NASH THEORY
multi objective optimization

↓

mono objective optimization

↓

GRADIENT METHOD

⇅

RESPONSE SURFACES

TEST

$$Test1(\mathbf{x}) = 1 + (A_1 - B_1)^2 + (A_2 - B_2)^2$$

$$Test2(\mathbf{x}) = (b_1 + 3)^2 + (b_2 + 1)^2$$

with $A_i = \sum_{j=1}^{2} (a_{ij} \sin(a_j) + b_{ij} \cos(a_j))$ for i=1,2

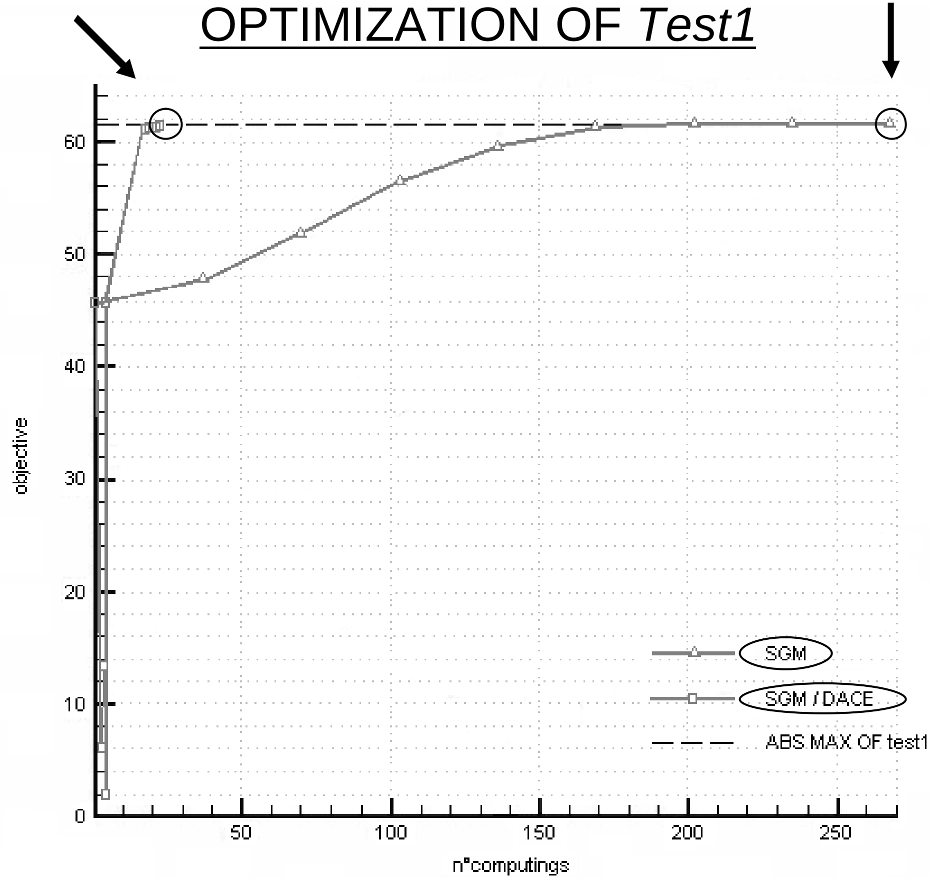
$B_i = \sum_{j=1}^{2} (a_{ij} \sin(b_j) + b_{ij} \cos(b_j))$ for i=1,2

$$\mathbf{a} = \begin{pmatrix} 0.5 & 1.0 \\ 1.5 & 2.0 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -2.0 & -1.5 \\ -1.0 & -0.5 \end{pmatrix}$$

$$a = (1.0, 2.0) \qquad b = \left(\sum_{i=1}^{n/2} x_i , \sum_{i=n/2+1}^{n} x_i \right)$$

where $\mathbf{x} \in \left[-\frac{p}{n/2}, \frac{p}{n/2} \right]^n$ and $n = 16$

OPTIMIZATION OF *Test1*



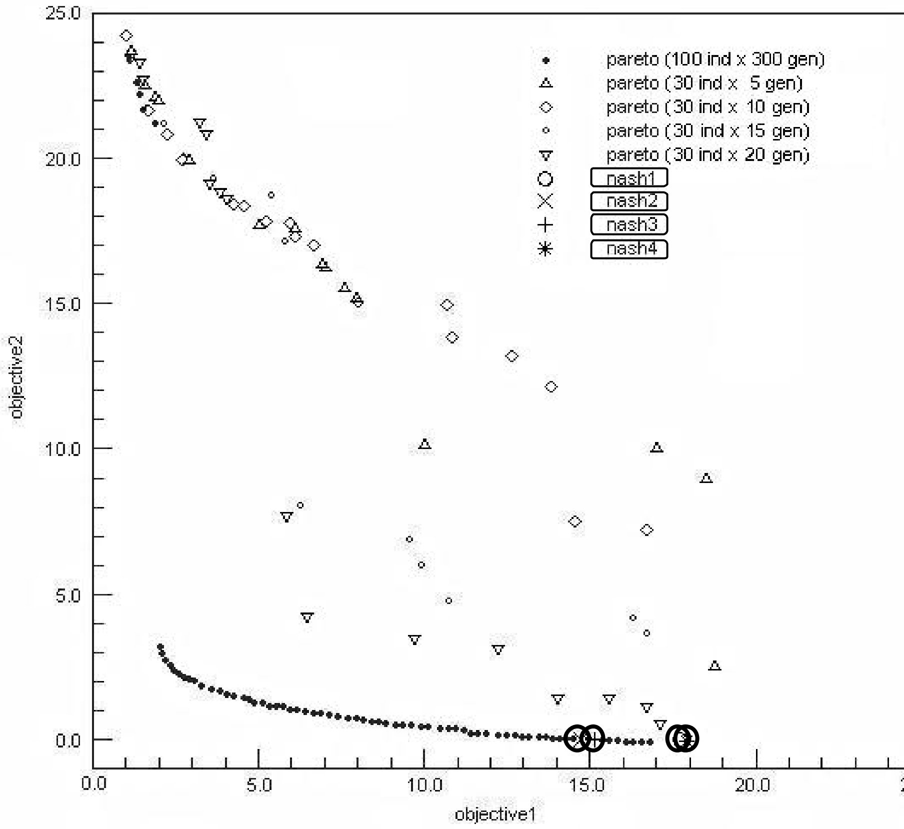
TEST

<u>MULTI OBJECTIVE MINIMIZATION</u>

<u>OF *Test1* AND *Test2*</u>

	σ	init.data set DACE	Test1	Test2	n°exchanges	n°computations
nash1	5	3	17.870	0.045	16	356
nash2	3	3	14.735	0.025	14	246
nash3	2	3	15.132	0.016	24	322
nash4	3	10	17.982	0.007	6	128

Comparison between Nash equilibrium and Pareto frontier computed by MOGA (Multi Objective Genetic Algorithm)



APPLICATION

DESIGN OF A TRANSONIC AIRFOIL IN STOCHASTIC OPERATIVE CONDITIONS

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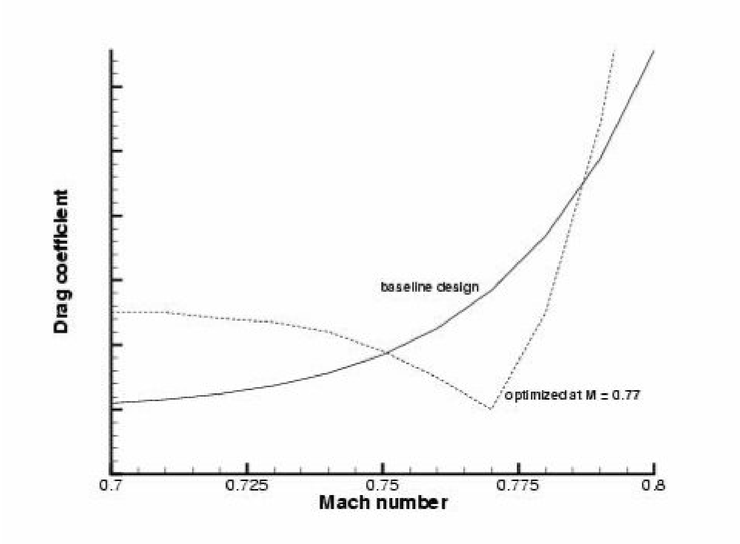
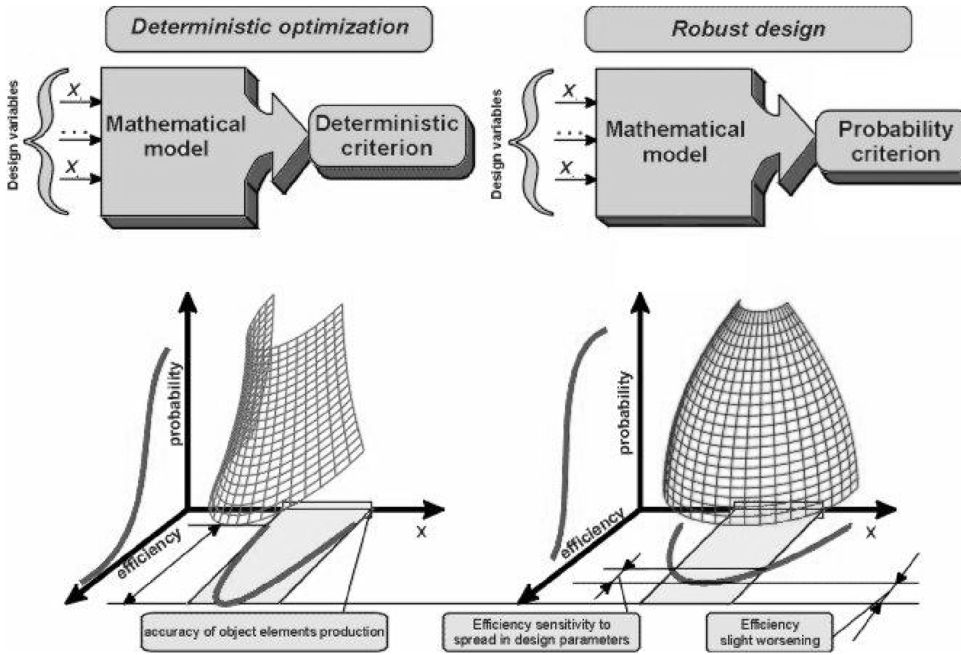
APPLICATION

DETERMINISTIC DESIGN:

OVER-OPTIMIZATION





COMPARISON BETWEEN DETERMINISTIC DESIGN AND PROBABILISTIC DESIGN

ROBUST DESIGN

$$F(xy):\Re^{n+m}\rightarrow\Re$$ function to optimize

where

x deterministic variables

y stochastic variables

- OPTIMIZATION OF **EXPECTED VALUE**

$$\bar{f}(x)=\frac{1}{n}\sum_{i=1}^{n} f_i$$

- MINIMIZATION OF **STANDARD DEVIATION**

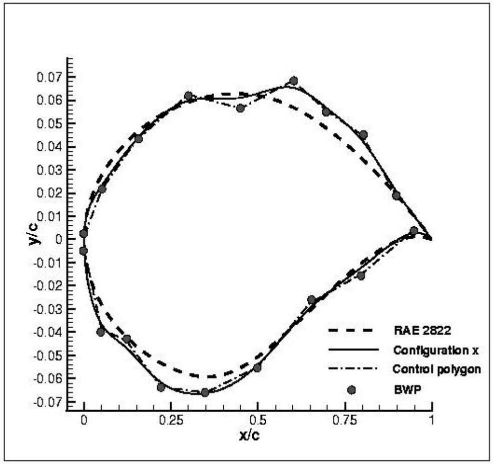
$$s_f(x)=\sqrt{\frac{\sum_{i=1}^{n}(f_i-\bar{f})^2}{n-1}}$$

ROBUST DESIGN

STOCHASTIC VARIABLES

- ANGLE OF ATTACK $a = 2^{\circ} \pm 0.5^{\circ}$
- MACH NUMBER OF FREE STREAM $M_{\infty} = 0.73 \pm 0.05$

DETERMINISTIC VARIABLES



OBJECTIVES AND CONSTRAINTS

$$\begin{cases} \min \overline{C}_d \\ \min s_{Cd} \end{cases}$$

with

$$\begin{cases} \overline{C}_d \le \overline{C}_{dRAE} \\ s_{Cd} \le s_{CdRAE} \\ \overline{C}_l \ge \overline{C}_{lRAE} \\ s_{Cl} \le s_{ClRAE} \\ t_{\max} \ge t_{\max RAE} \end{cases}$$

ROBUST DESIGN

$$\bar{f}(x)=\frac{1}{n}\sum_{i=1}^{n} f_i$$

$$s_f(x)=\sqrt{\frac{\sum_{i=1}^{n}(f_i-\bar{f})^2}{n-1}}$$

COMPUTING OF $\overline{C}_d, s_{Cd}, \overline{C}_l$ AND s_{Cl}

COMPUTATIONALLY and TIME EXPENSIVE

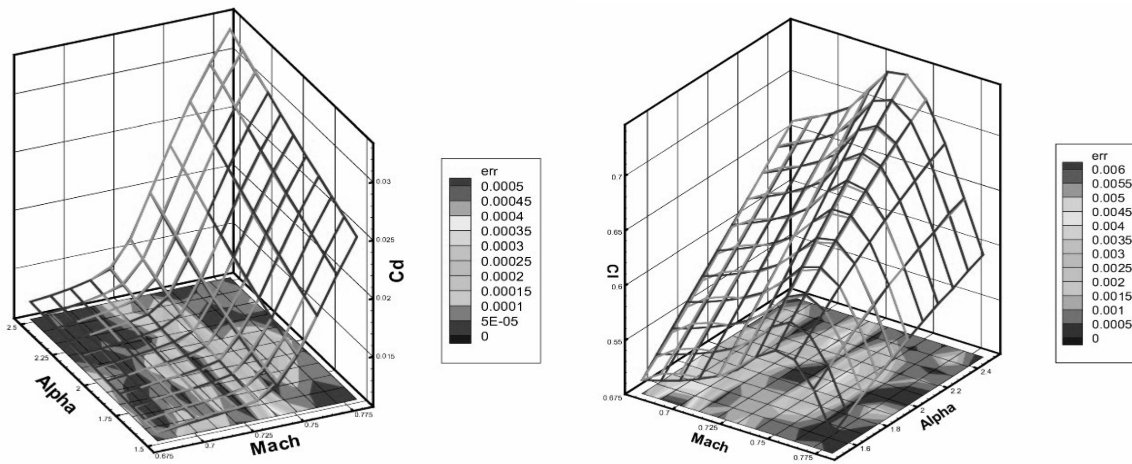
[redacted]

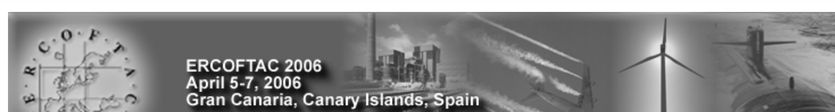
DACE MODEL OF C_d AND C_l

$Re = 1.5E06$

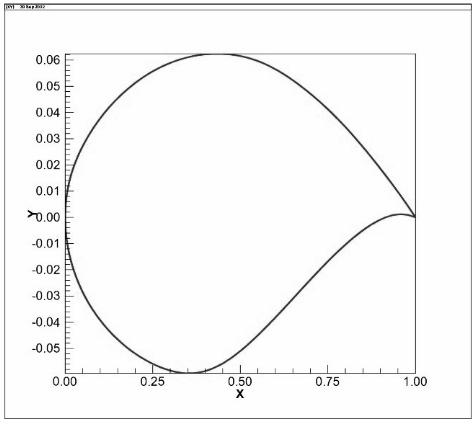
$M_\infty = 0.68 \div 0.78$

$a = 1.5° \div 2.5°$

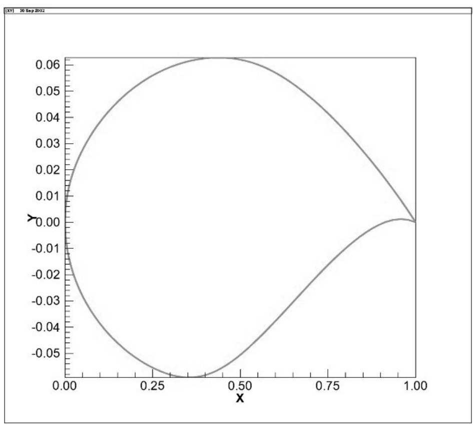




ROBUST DESIGN



OPTIMIZED AIRFOIL



RAE2822

EXPLORATIVE DESIGN

- 4 exchanges of variables between players
- 87 fluid dynamics simulations
- Nash frequency equal to 2

	optimized airfoil	**RAE2822 airfoil**
$\overline{C}_d$	0.017351	0.017297
s_{C_d}	0.006128	0.006087
$\overline{C}_l$	0.604603	0.610551
s_{C_l}	0.053835	0.053760
$t_{\max}$	0.121223	0.121294

Comparison between objectives of optimized airfoil (explorative design) and RAE2822.

ROBUST DESIGN

CONSERVATIVE DESIGN

	optimized airfoil	RAE2822 airfoil
$\overline{C}_d$	0.017133	0.017297
s_{C_d}	0.005996	0.006087
$\overline{C}_l$	0.608349	0.610551
s_{C_l}	0.053552	0.053760
$t_{\max}$	0.121334	0.121294

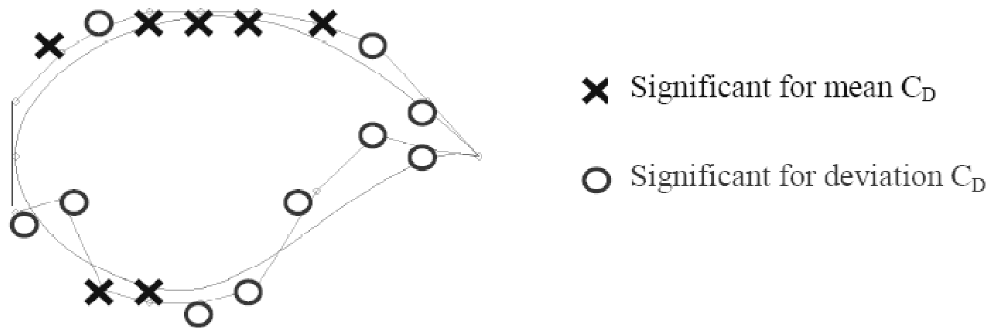
- 3 exchanges of variables between players
- 134 fluid dynamics simulations
- Nash frequency equal to 3

Comparison between objectives of optimized airfoil (conservative design) and RAE2822.

ROBUST DESIGN

REMARKS

- Cause of constant step lenght of SGM difficult to find solutions inside constraints.
- Choice of domain space decomposition is very important for the efficiency of optimization as it has been demonstrated in “Application of Game Strategy in Multi-objective Robust Design Optimisation Implementing Self-adaptive Search Space Decomposition by Statistical Analysis”, A.Clarich, V.Pediroda, L.Padovan, C.Poloni, J. Periaux, (2004), *European Congress on Computational Methods in Applied Sciences and Engineering* ECCOMAS 2004. An adaptive strategy for variables distribution is needed.



CONCLUSION

- IMPLEMENTATION OF A NASH THEORY WITH GRADIENT BASED OPTIMIZATION ALGORITHM FOR MULTI OBJECTIVE PROBLEMS

- SGM TO OPTIMIZE EACH ONE OF OBJECTIVES

- ADAPTIVE DACE MODEL, A PARTICULAR EFFICIENT EXTRAPOLATION METHOD, TO CALCULATE THE DERIVATIVES REQUIRED BY SGM

❖ TESTS ON MATHEMATICAL FUNCTIONS

❖ APPLICATION TO DESIGN UNDER UNCERTAINTIES OF AN AIRFOIL IN TRANSONIC FIELD

CONCLUSION

FURTHER STEPS

• in mathematical test cases great efficiency : quick and accurate, but limits due to lack of adaptivity to different problems.
In particular
- constrained problems require more investigation
- introduction of a criteria for the re-distribution of variables to players

• in Robust Design application:
- explorative design has been really successful: we can think of using Nash/SGM to investigate an unknown problem, with the advantage to get quickly an optimum point, and next, starting from the Nash equilibrium, to obtain more accurate solutions by Pareto Games