A Fast Numerical Optimization Algorithm for Aircraft Continuous Descent Approach

J.M. Canino*, J. González and L. Gómez

*Researcher and private pilot with commercial license
CONTENTS:

INTRODUCTION
AIRCRAFT MODEL
DESCENT STRATEGIES
NUMERICAL SOLUTION SCHEME
RESULTS
CONCLUSIONS AND FUTURE WORK
INTRODUCTION

Airspace congestion near to airports is a critical problem, causing costly ground delays and environmental problems (noise).

The continuous traffic increase is pushing the current system to its limits... and it keeps growing...

There is a need to fully reduce noise and environmental impact through optimizing aircraft traffic specially near to airports. Thus means, finding/designing new techniques for approaching to runway.

WE FOCUS ON OPTIMIZING THE VERTICAL DESCENT PROFILE IN THE FINAL APPROACH TO AIRPORT
METHODOLOGY

AIRCRAFT MOTION MODEL

HORIZONTAL/VERTICAL TRAJECTORIES

OPTIMIZATION

ALTIMETRY

F(X)

x

t
AIRCRAFT MODEL

- Based on the standard set of simplified equations


\[
\begin{align*}
\frac{ds}{dt} &= v_G = u_W \cdot \cos(\delta_G - \delta_W) + v_T \cdot \cos \left( \arcsin \left( \frac{u_W \cdot \sin(\delta_G - \delta_W)}{v_T} \right) \right) \\
\frac{dh}{dt} &= \gamma_a \cdot v_T = \gamma_i \cdot v_G \\
\frac{dv_T}{dt} &= \left[ \frac{T - D}{m} \right] - g \gamma_a - \gamma_a v_T \cdot \frac{du_W}{dh}
\end{align*}
\]

s: horizontal path length
h: geometrical altitude
v_T: true airspeed
v_G: ground speed
T: thrust
D: drag

\(u_W\): wind speed
\(v_G\): ground speed
\(\gamma_a\): aerodynamic flight angle
\(\gamma_i\): inertial flight path angle
\(\delta_G\): direction of ground speed
\(\delta_W\): angle of the wind speed
m: aircraft mass
DESCENT STRATEGIES:

Standard Vertical Step Descent Profile

The trajectory is composed of a series of segments keeping two of the following variables constant:

- Engine control (idle thrust / maximum thrust)
- Speed (Mach / CAS)
- Vertical rate (altitude rate / inertial flight path angle)

The values of the constant variables are specified to be

- within the constraints of air traffic control
- performance limits of the aircraft (BADAS database)

IT REPRESENTS THE NORMALLY USED DESCENT APPROACH FOR COMMERCIAL/ CIVIL FLIGHTS
DESCENT STRATEGIES:

Continuous Vertical Descent Profile

CDA (Continuous Descent Approach) was originally developed as a procedure for efficient methodology for reducing the noise (and fuel) of approaching aircraft close to airports.

The CDA technique results in lower noise levels on the ground through two main effects:

- CDA flight-path is always higher,
- keeping the aircraft on a continuous descent, the overall engine power levels are kept lower, causing less noise than if the aircraft were required to fly level.

IT BEING USED AS AN EXPERIMENTAL DESCENT APPROACH FOR COMMERCIAL/ CIVIL FLIGHTS WHEN AIR TRAFFIC ALLOWS IT
DESCENT STRATEGIES:

Standard Step Descent Profile/ CDA Descent Profile

TOD (CDA)    TOD (Step Descent)

Altitude (ft.)

0  10  20  30  40  50  60  70  80

Distance to Threshold (NM)

0  2000  4000  6000  8000  10000  12000

NO LEVEL FLIGHT

LEVEL FLIGHT:

- HIGH FUEL/NOISE
- SATISFY RUNWAY CAPACITY

Step Descent
CDA
**OUR PROPOSAL:**

**STANDARD VERTICAL STEP DESCENT:**
- It handles the actual traffic flow requirements
- Non-optimal/high noise/high fuel consumption

**CONTINUOUS DESCENT APPROACH:**
- Low noise/low consumption
- Problems to account for the actual traffic flow requirements

Objective: to fit a desired arrival time avoiding level flight

Variables: Speed rate (flap configuration/flight path angle)
NUMERICAL SOLUTION SCHEME

Minimize $f(x)$

Subject to $g_i(x) \leq 0$ for $i = 1, \ldots, m$

Set of allowed nominal speeds

Time constraint

Cost Function: Time to destination (aerodynamic)

$V_{S1} \leq V_{S1} \leq V_{\text{max} S1}$

$V_{S2} \leq V_{S2} \leq V_{\text{max} S2}$

$V_{Sn} \leq V_{Sn} \leq V_{\text{max} Sn}$

X ? X

Speeds/flap configuration/flight path angle
NUMERICAL SOLUTION SCHEME:

**Characteristics of the stated problem:**

- Cost function $f(x)$ is not known explicitly, neither the gradient; 
  
  *they must be evaluated through a simulated program*

- Cost function is not convex (and nonlinear)

*The problem can be classified as a nonlinear programming problem*
NUMERICAL SOLUTION SCHEME:

- Genetic
- Simulated annealing
- Monte Carlo ...

POSSIBILITIES

CLASSICAL METHODS

- using derivatives (gradient like)
- non using derivatives

ROSENBROCK’S METHOD
NUMERICAL SOLUTION SCHEME:

ROSENBROCK’S METHOD

- It is a classical multidimensional deterministic optimization method from 1960’s
- It does not use gradient information (only function evaluations)

It is easy to program and..... Seems to work well in our problem!!!!
As an example, we plot the map for the function

\[ f(x_1, x_2) = (x_1 - 2)^4 + (x_1 - 2x_2)^2 \]

Minimum is at (2,1)

Progress in both variables

Rotate axis by Gram-Schmidt orthogonalization procedure considering the improvement reached on each variable to estimate the new coordinate axis
ROSENROCK’S METHOD

More efficient.... They need gradient information
Flow-chart of the implemented algorithm

- Set of specific parameters
- Set of allowed nominal descent speeds
- VERTICAL DESCENT PROFILE $(t_{\text{estimated},})$
- Second order Runge-Kutta numerical integration
- BADAS

Set of specific algorithm parameters
- NUMERICAL NONLINEAR OPTIMIZATION
- GRAM-SCHMIDT ORTHOGONALIZATION
- $\|t - t_{\text{estimated}}\| < e_1$

- $x_0$: arrival time, stop criteria

Solution $(v_{dL1}, ..., v_{dLn})$
RESULTS

Simulating horizontal/vertical trajectories

Desired (blue), real (red)
RESULTS

Simulating horizontal/vertical trajectories

\[ V_{nom}, V_{tas}, V_{cas} \quad \text{(blue, red, green)} \]

- Speed (Knots)
  - 500
  - 450
  - 400
  - 350
  - 300
  - 250

- Distance (meters)
  - 0
  - 1
  - 2
  - 3
  - 4
  - 5
  - 6
  - \( x \times 10^5 \)
RESULTS

CONTINUOUS DESCENT PROFILE

Layer 3, $v = v_3$  $v_{3\text{min}} < v_3 < v_{3\text{max}}$

Layer 2, $v = v_2$  $v_{2\text{min}} < v_2 < v_{2\text{max}}$

Layer 1, $v = v_1$  $v_{1\text{min}} < v_1 < v_{1\text{max}}$

No level flight
RESULTS

CONTINUOS DESCENT PROFILE

- **LAYER1**: 0 to 2000 ft. \( V_{\text{layer1}_\text{min}} < V < V_{\text{layer1}_\text{max}} \)
- **LAYER2**: 2000 to 6000 ft. \( V_{\text{layer2}_\text{min}} < V < V_{\text{layer2}_\text{max}} \)
- **LAYER3**: > 6000 ft. \( V_{\text{layer3}_\text{min}} < V < V_{\text{layer3}_\text{max}} \)

Two different solutions (non convexity)

Adding fuel consumption allows to eliminate one of them
CONCLUSIONS

• A EN-ROUTE AND DESCENT TRAJECTORY AIRCRAFT SIMULATOR HAS BEEN DEVELOPED

• CODE IS IMPLEMENTED IN MATLAB/C++ AND THE COMPUTATIONAL COST IS LOW ENOUGH TO ALLOW ITS INTEGRATION IN AN AUTOMATIC TRACKING TOOL TO ASSIST USERS (PILOTS)

• AN OPTIMIZATION PROBLEM HAS BEEN DEFINED IN ORDER TO DEAL WITH THE CONTINUOUS DESCENT APPROACH AND RESULTS SHOWS THAT FITTING A TIME-TO-DESTINATION IS REACHED FOR THE CASES UNDER CONSIDERATION
FUTURE WORK

• EXTEND THE OPTIMIZATION PROBLEM TO INVOLVE FUEL CONSUMPTION

• INTEGRATE THE PROPOSED OPTIMIZATION METHODOLOGY INTO A FLIGHT SIMULATOR WITH AN EASY GRAPHIC INTERFACE (OR COMMERCIAL/OPEN SOFTWARE)

Actual Flight-Simulator Environment linking to the trajectory software, but not to the optimization methodology.... Yet!
A Fast Numerical Optimization Algorithm for Aircraft Continuous Descent Approach

J.M. Canino*, J. González and L. Gómez
CTM, ULPGC

* Researcher and private pilot with commercial license