Design Optimization of High Speed Inlets

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I. Introduction

A high (supersonic) speed inlet provides the required airflow to the ramjet or turbojet engine with minimum distortion over a range of flight Mach numbers and vehicle angle of attack and sideslip. High speed inlets may be categorized as external compression (Fig. 1) or mixed compression (Fig. 2). In the former case, an oblique shock system forms upstream of the cowl leading edge, followed by a terminal (normal) shock located at the cowl. This is the most common design for inlets used in supersonic aircraft. In the latter case, the shock system is both external (i.e., upstream of the cowl leading edge) and internal (i.e., downstream of the cowl leading edge) and terminated by a normal shock. The average Mach number at the geometric throat (i.e., the location of the minimum cross sectional area) is low supersonic.

Figure 1. External compression

Figure 2. Mixed compression

II. Design Criteria

The quality of the aerodynamic design of a high speed inlet is determined by a variety of different criteria. The most common criterion is the maximization of total pressure recovery coefficient $\eta$ defined by

$$\eta = \frac{\bar{p}_t}{p_{t,\infty}}$$

where $\bar{p}_t$ is the area-averaged total pressure at the outflow of the subsonic diffuser and $p_{t,\infty}$ is the freestream total pressure. For a perfectly isentropic deceleration, $\eta$ would be unity; however, viscous dissipation and shock waves (in supersonic inlets) result in a decrease in $\eta$.

The second criterion is the achievement of an adequate mass flow rate coefficient $\varepsilon$ defined by

$$\varepsilon = \frac{\dot{m}}{\dot{m}_m}$$

where $\dot{m}$ is the air mass flow rate into the inlet, and $\dot{m}_m$ is the maximum mass flow rate at zero angle of attack and sideslip. Both $\dot{m}$ and $\dot{m}_m$ depend on the inlet geometry and freestream Mach number. For supersonic inlets, the maximum mass flow rate occurs when the system of external compression waves (including shocks) is focused at the cowl leading edge (if indeed this condition can occur based upon the shape of the inlet) at some Mach number. This is denoted the design condition and the corresponding Mach number is denoted the design Mach number.
A third criterion, applicable to mixed compression inlets only, is the requirement that the internal shock system form at a specified Mach number (or range of Mach numbers). This is known as **inlet start**. Define the contraction ratio

\[ \chi = \frac{\text{cross sectional area at inlet entrance}}{\text{cross sectional area at inlet throat}} \] (3)

where the *throat* is the location of minimum cross sectional area\(^1\). The contraction ratio \( \chi \) must be less than the value necessary for sonic conditions at the throat. Using the area-averaged Mach number at the inlet entrance, a one-dimensional gasdynamics analysis is used to predict the Mach number at the throat. If the predicted throat Mach number is supersonic, then experience indicates that the internal shock system (downstream of the inlet entrance) will likely form and consequently the inlet will start.

A fourth criterion is the static aerodynamic stability margin (denoted the *static margin*) defined by

\[ \Delta s = x_{cp} - x_{cg} \] (4)

where \( x_{cp} \) and \( x_{cg} \) are the locations of the center of pressure and center of gravity, respectively, relative to the nose of the missile. The static margin must be positive for stability.

A fifth criterion is minimization of the *radar cross section* (radar signature). An air inlet modifies the shape of the air vehicle and consequently its radar signature as determined by the radar cross section \( \sigma \) defined by\(^1\)

\[ \sigma = \lim_{R \to \infty} \frac{4\pi R^2}{\mid \vec{E}_s \mid^2} \frac{\mid \vec{E}_i \mid^2}{\mid \vec{E}_s \mid^2} \] (5)

where \( \vec{E}_i \) and \( \vec{E}_s \) are the incident and scattered electric fields and \( R \) is the distance from the transmitter to the vehicle.

The first through third criteria listed above attempt to isolate the inlet optimization problem from the overall vehicle design optimization in order to simplify the task of inlet design. Ultimately, the overall performance of the vehicle during its mission define the design criteria. For example, Andreev and Penzin\(^2\) modeled vehicle performance over a mission profile to optimize the integration of a scramjet inlet with a hypersonic air vehicle.

### III. Single Point and Mission Design

The simplest design optimization problem for a high speed inlet is a single point, single criterion optimization corresponding to a fixed Mach number, altitude and vehicle orientation (*i.e.*, angles of attack and sideslip), and single criterion for optimization (*e.g.*, maximize \( \eta \)) with perhaps a single constraint (*i.e.*, \( \varepsilon > \varepsilon_o \)).

A more realistic design optimization problem for an air inlet is the *mission design* wherein performance of the air inlet over a range of Mach numbers, altitudes and vehicle orientations is considered. This concept is illustrated in Fig. 3 for an airbreathing supersonic missile. During the initial phase of the mission, the missile accelerates to a specified Mach number and altitude. In the second phase, the missile cruises at supersonic speed towards the target. In the third and final phase, the missile maneuvers to acquire the target. In each phase, the specific flight conditions (*i.e.*, Mach number, altitude, angles of attack and sideslip) vary with time, and the design requirements differ.

For example, during the initial phase (acceleration), the mass flow rate coefficient \( \varepsilon \) must exceed a critical value in order to provide adequate mass flow to the combustor. During the cruise phase, the total pressure

\(^1\)The cross-sectional area is typically measured in a plane which is approximately perpendicular to the lower inner surface (centerbody, see Figs. 1 and 2) and upper inner surface (cowl).
recovery coefficient must be maximized in order to achieve the maximum range of the missile. Typically, the mission profile is approximated by one or more flight points (defined by Mach number, altitude, and angles of attack and sideslip) in each phase.

IV. Pareto Set

These multiple design objectives can be expressed as minimizations of a set of functions\(^ii\). The Multicriteria Design Optimization (MDO) problem is therefore

\[
\min_{x \in D} f_j(x) \quad \text{for} \quad j = 1, \ldots, m
\]

where \(x\) is the vector of design variables, and subject to a set of constraints

\[
d_k \leq 0 \quad \text{for} \quad k = 1, \ldots, l
\]

In general, the objective functions \(f_j\) may be conflicting, \(i.e.,\) decreasing the value of one objective function may lead to increasing the value of one or more other objective functions. Thus, there may be (in general) no single best design. Consequently, it is necessary to extend the notion of an optimal (best) design. An example is shown in Fig. 4 assuming \(m = 2\). The values of the objective functions \(f_1\) and \(f_2\) are shown for five different designs (denoted as 1 to 5) where each design is represented by a single point in the \((f_1, f_2)\) plane. Design 2 is clearly superior to design 4 (\(i.e.,\) design 2 has a lower value of both \(f_1\) and \(f_2\)). Likewise, design 3 is superior to design 5 (\(i.e.,\) design 3 has a lower value of both \(f_1\) and \(f_2\)). However, designs 1, 2 and 3 are equivalent, since none of them is superior to any of the others in both design objectives \(f_1\) and \(f_2\). The three designs 1, 2 and 3 form the Pareto Set of optimal designs, and represent the solution to the design optimization problem. In otherwords, the “solution” to the Multicriteria Design Optimization problem is a set of designs which are “equivalent”. There is no single “best” design.

V. Example

An example of a Multicriteria Design Optimization for the aerodynamic design of a high speed inlet is presented to illustrate the capability for complex design optimization.

A. Configuration

Gaiddon, Knight and Poloni\(^3\) performed a fully automated Multicriteria Design Optimization of an inlet for a ramjet-powered missile for a mission using a variety of optimization algorithms. The missile, shown in Fig. 5, is intended to fly at a positive angle of attack during most of the mission, and therefore a single inlet is mounted underneath the missile body to take advantage of the precompression generated by the body.

The inlet has a rectangular cross-section (Fig. 6). The design variables are the ten parameters that define the inlet geometry, namely, the lengths and angles of the segments of the ramp \((L_1, L_2, \theta_1, \theta_2)\) and cowl \((L_{c1}, L_{c2}, \theta_{c1}, \theta_{c2})\), the relative axial position of the cowl apex from the ramp apex \((x_{cowl})\) and the capture area \((y_{cowl} \times \text{width})\). The geometrical characteristics of the missile and range of the parameters are listed in Tables 1 and 2, respectively.

\(^{ii}\)Maximizing a function is equivalent to minimizing its negative, so we may assume all functions are to be minimized without loss of generality.
Table 1. Geometric Characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>3.2 m</td>
</tr>
<tr>
<td>Caliber</td>
<td>250 mm</td>
</tr>
<tr>
<td>$S_R$</td>
<td>0.049087 $m^2$</td>
</tr>
<tr>
<td>Wing area</td>
<td>$1.6S_R$</td>
</tr>
<tr>
<td>Wing span</td>
<td>500 mm</td>
</tr>
<tr>
<td>Fin area</td>
<td>$0.75S_R$</td>
</tr>
<tr>
<td>Capture area</td>
<td>$0.45S_R$</td>
</tr>
<tr>
<td>Base area</td>
<td>$0.28S_R$</td>
</tr>
</tbody>
</table>

Table 2. Range of Design Variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1, L_2$ (mm)</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>$\theta_1, \theta_2$</td>
<td>0°</td>
<td>30°</td>
</tr>
<tr>
<td>$L_{c_1}, L_{c_2}$ (mm)</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>$\theta_{c_1}, \theta_{c_2}$</td>
<td>0°</td>
<td>30°</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>$x_{cowl}$ (mm)</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>$y_{cowl}$ (mm)</td>
<td>50</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 3. Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{II}$</td>
<td>thrust coefficient</td>
</tr>
<tr>
<td>$C_A$</td>
<td>axial force coefficient</td>
</tr>
<tr>
<td>$C_N$</td>
<td>normal force coefficient</td>
</tr>
<tr>
<td>$m$</td>
<td>vehicle mass</td>
</tr>
<tr>
<td>$N_x$</td>
<td>horizontal acceleration factor</td>
</tr>
<tr>
<td>$N_z$</td>
<td>vertical acceleration factor</td>
</tr>
<tr>
<td>$q_\infty$</td>
<td>freestream dynamic pressure</td>
</tr>
<tr>
<td>$S_R$</td>
<td>reference area</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle of attack</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>injected fuel equivalence ratio</td>
</tr>
</tbody>
</table>

B. Flight Performance

The flight performance of the missile was determined by the evaluating the aerodynamic and propulsive forces on the vehicle along the air axes (i.e., horizontal and vertical axes),

\[
(C_{II} - C_A) \cos \alpha - C_N \sin \alpha = N_x \frac{mg}{q_\infty S_R} \quad (8)
\]

\[
C_N \cos \alpha + (C_{II} - C_A) \sin \alpha = N_z \frac{mg}{q_\infty S_R} \quad (9)
\]

where the terms are defined in Table 3. By convention, $N_x = 0$ and $N_z = 1$ in balanced flight condition. Note that the axial force coefficient $C_A$ and normal force coefficient $C_N$ are functions of the missile angle of attack $\alpha$, and thrust coefficient $C_{II}$ is a function of both the angle of attack $\alpha$ and the fuel equivalence ratio $\phi_i$.

For an acceleration flight condition, the horizontal acceleration factor $N_x$ is maximized at level flight ($N_z = 1$). Thus,

\[
C_N \cos \alpha + (C_{II} - C_A) \sin \alpha = \frac{mg}{q_\infty S_R} \quad (10)
\]

The ramjet is set for maximum thrust and is limited by either the inlet total pressure recovery or maximum burnt equivalence ratio $\phi_b = 1$. The balanced flight angle of attack is obtained from (10), and the acceleration...
factor $N_z$ is obtained from (8),

$$N_z = \frac{q_{\infty} S_R}{mg} (C_H - C_A) \cos \alpha - C_N \sin \alpha \quad (11)$$

For a maneuver flight condition, the vertical acceleration factor $N_z$ is maximized subject to $N_x = 0$. From (8),

$$\tan \alpha = \frac{(C_H - C_A)}{C_N} \quad (12)$$

Maximum thrust is assumed. The vertical acceleration factor $N_z$ is obtained from (9),

$$N_z = \frac{q_{\infty} S_R}{mg} (C_N \cos \alpha + (C_H - C_A) \sin \alpha) \quad (13)$$

For a cruise condition, the fuel consumption is minimized subject to $N_x = 0$ and $N_z = 1$. Thus, from (8) and (9),

$$C_N(\alpha) = \left( \frac{mg}{q_{\infty} S_R} \right) \cos \alpha \quad (14)$$

yields the angle of attack $\alpha$ by iteration. From (8),

$$C_H = C_A + C_N \tan \alpha \quad (15)$$

yields the value of thrust required and thus the fuel consumption from the ramjet model.

### C. Design Objectives

A Multicriteria Design Optimization of the inlet geometry was performed for the mission comprised of three flight points as described in Table 4 and specified design objectives at each flight point as detailed in Table 5. The first flight point is acceleration at Mach 1.9. The objective is maximizing the horizontal acceleration coefficient $N_x$ while maintaining level flight ($N_z = 1$). There is no constraint on the fuel consumption. The second flight point is cruise at Mach 2.8. There is no horizontal ($N_x = 0$) or vertical ($N_z = 1$) acceleration, and the objective is to minimize the fuel consumption rate $m_k$ in order to maximize the missile range. The third flight point is maneuver at Mach 2.2. The objective is to maximize the vertical acceleration coefficient $N_z$ (to avoid defensive countermeasures and contact the target) without horizontal acceleration ($N_x = 0$), and without constraint on the fuel consumption.

A preliminary trade study\textsuperscript{3} indicated that the three design objectives were conflicting, and hence a Pareto Set of optimal designs would be expected. For example, a single criterion design optimization based upon the cruise flight point yielded a design which decelerated for the acceleration flight point.

#### Table 4. Mission Description

<table>
<thead>
<tr>
<th>Flight Point</th>
<th>Mach</th>
<th>Altitude (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>1.9</td>
<td>100</td>
</tr>
<tr>
<td>Cruise</td>
<td>2.8</td>
<td>15,000</td>
</tr>
<tr>
<td>Maneuver</td>
<td>2.2</td>
<td>100</td>
</tr>
</tbody>
</table>

**LEGEND**

- $N_x$ Acceleration in horizontal direction (in g’s)
- $N_z$ Acceleration in vertical direction (in g’s)
- $m_k$ fuel consumption (kg/s)

#### Table 5. Design Objectives

<table>
<thead>
<tr>
<th>Flight Point</th>
<th>$N_x$</th>
<th>$N_z$</th>
<th>$m_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>max $N_x$</td>
<td>$N_z = 1$</td>
<td>none</td>
</tr>
<tr>
<td>Cruise</td>
<td>$N_x = 0$</td>
<td>$N_z = 1$</td>
<td>min $m_k$</td>
</tr>
<tr>
<td>Maneuver</td>
<td>$N_x = 0$</td>
<td>max $N_z$</td>
<td>none</td>
</tr>
</tbody>
</table>

**LEGEND**

- $N_x$ Acceleration in horizontal direction (in g’s)
- $N_z$ Acceleration in vertical direction (in g’s)
- $m_k$ fuel consumption (kg/s)
D. Design Optimization Methodology

Fig. 7 shows the automated design optimization sequence. At every iteration of the sequence, a specific inlet geometry is selected by the Multicriteria Optimizer. The inlet geometry is evaluated to insure that geometrical constraints are not violated (i.e., the cowl and ramp do not intersect each other). If a geometrical constraint is violated, the geometry is rejected and the control flow is returned to the Multicriteria Optimizer to generate a new geometry. If no geometrical constraints are violated, a three-dimensional flow grid is generated. Then, the flowfield is computed for the three flight points using the hybrid code 2ES3D developed by Blaize and Bourdeau et al. The internal aerodynamic coefficients of the inlet are computed from the converged flowfield, and combined with previously computed external aerodynamic coefficients into a missile trajectory analysis incorporating a separate ramjet model. The baseflow region is not computed, and the base drag is estimated using a semi-empirical correction.

The missile performance (i.e., horizontal and vertical acceleration coefficients $N_x$ and $N_z$ and rate of fuel consumption $m_k$) are computed for each of the three mission points and the results returned to the Multicriteria Optimizer. The procedure is repeated until convergence, i.e., an acceptable represent of the Pareto Set is obtained. Several different optimization algorithms were evaluated using the software environment modeFRONTIER. They included the Nelder-Mead Simplex Method, Genetic Algorithms (see, for example, Deb), and several variants of Evolutionary Strategies (see, for example, Bäck).

E. Results

The results of the optimization using the Genetic Algorithm are presented in Figs. 8 to 10. Each figure is a projection of the three-dimensional Pareto Set in the objective functions $N_x$ (the horizontal acceleration in $g$’s at the acceleration flight point at Mach 1.9 and 100 m altitude), $N_z$ (the vertical acceleration in $g$’s at the maneuver point at Mach 2.2 and 100 m altitude) and $m_k$ (the rate of fuel consumption at the cruise point at Mach 2.8 and 15 km altitude) onto a two-dimensional surface.

The Pareto Set contains 133 different designs. Each design represents a different inlet geometry with different performance values for each of the three design criteria listed in Table 5. Consider the projection of the Pareto Set on the $N_x$ vs $N_z$ plane as shown in Fig. 8. The vertical axis (“Maneuver”) is $N_z$ and the horizontal axis (“Acceleration”) is $N_x$. Each symbol in the figure represents a different inlet geometry. For most of these designs, an improvement in horizontal acceleration $N_x$ also yields an improvement in vertical acceleration $N_z$. However, there is a region ($N_x > 0.288$) where improvement in horizontal acceleration results in a decrease in vertical acceleration. Moreover, Fig. 9 shows that virtually all improvements in horizontal acceleration $N_x$ result in an increase in fuel consumption $m_k$ at the cruise point. Thus, there exists a clear tradeoff between acceleration (at the acceleration point) and fuel consumption (at the cruise point) in selecting the actual inlet geometry.

The convergence properties of the Genetic Algorithm are illustrated in Fig. 11 which displays the history of evolution of $N_x$ (acceleration point). The non-monotonic behavior is typical of Genetic Algorithms in Multicriteria Design Optimization.
Figure 8. $N_x$ vs $N_x$

Figure 9. $M_k$ vs $N_x$

Figure 10. $M_k$ vs $N_x$

Figure 11. $N_x$ optimization
VI. Acknowledgments

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References


