THERMOFLUIDDYNAMICS: DO WE SOLVE THE RIGHT KIND OF EQUATIONS?

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ABSTRACT

Direct numerical simulations of compressible fluid flows with heat and mass transfer can be carried out these days, using the Navier-Stokes equations at least for low Re-number flows (Re ≤ 50,000). Compressibility is taken into account by the "compressible form" of the continuity equations. Temperature and density or pressure variations in space are accounted for by the dependences of the molecular transport coefficient on the local thermodynamic state, i.e. by introducing the viscosity coefficient $\mu(T, P)$, the thermal conductivity coefficient $\lambda(T, P)$ and the mass diffusion coefficient $D(T, P)$. Hence, the following equations are solved:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho U_i)}{\partial x_i} = 0 \qquad (1) \qquad\qquad \rho\left[\frac{\partial U_j}{\partial t} + U_i \frac{\partial U_j}{\partial x_i}\right] = -\frac{\partial P}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_i} + \rho g j \qquad (2)$$

$$\rho\left[\frac{\partial e}{\partial t} + U_i \frac{\partial e}{\partial x_i}\right] = -\frac{\partial \dot{q}_i}{\partial x_i} - P\frac{\partial U_j}{\partial x_j} - \tau_{ij}\frac{\partial U_j}{\partial x_i} \qquad (3)$$

with the following transport terms:

$$\dot{m}_i = 0 \qquad (4) \qquad\qquad \dot{q}_i = -\lambda \frac{\partial T}{\partial x_i} \qquad (5)$$

and

$$\tau_{ij} = -\mu\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right) + \frac{2}{3}\delta_{ij}\mu\frac{\partial U_k}{\partial x_k} \qquad (6)$$

For strong temperature and density gradients, derivations of the diffusive transports of heat and mass yield additional terms known as Soret term for mass diffusion and Dufour term for heat diffusions. To derive these terms for density and temperature gradients or for pressure and temperature gradients, a particular methodology is developed and is applied. It is extended to also derive the additional diffusion caused momentum transport term yielding the complete τ_{ij}- term to read:

$$\tau_{\mathrm{ij}} = -\nu\left[\frac{\partial(\rho U_j)}{\partial x_i} + \frac{\partial(\rho U_i)}{\partial x_j}\right] + \frac{2}{3}\delta_{ij}\nu\left(\frac{\partial(\rho U_k)}{\partial x_k}\right) - \frac{\nu}{2T}\left[\rho U_j \frac{\partial T}{\partial x_i} + \rho U_i \frac{\partial T}{\partial x_j}\right] \qquad (7)$$

The latter terms represent a temperature gradient driven momentum transport that can be important in thermo fluid dynamics when strong temperature gradient exist.

Hence, the paper suggests to take the derived complete relationships for τ_{ij}, $\dot{q}_i$ und $\dot{m}_i$ into account when treating compressible flows with heat and mass transfer with strong thermodynamic fluid property gradients. The additional terms become very important when high velocity flows are treated. They represent non-linear terms also yielding contributions to the momentum transport in the Reynolds equations. It is shown that they make quantities like the turbulent Prandtl-number dependent on the Re-number and the fluid Prandtl-number.