Scale-Adaptive Simulation für Technische Strömungen

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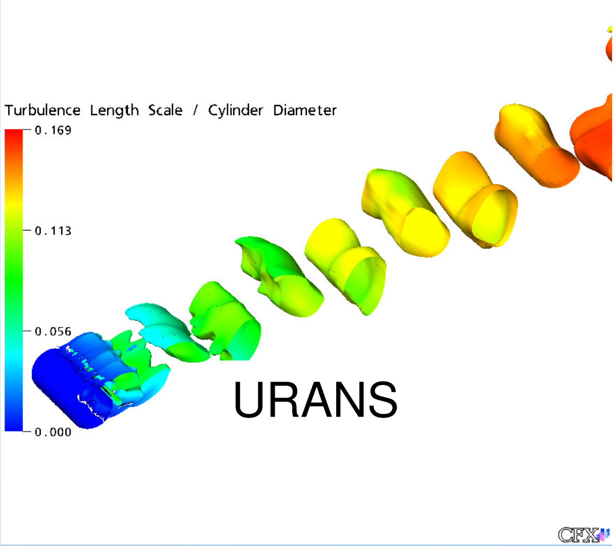
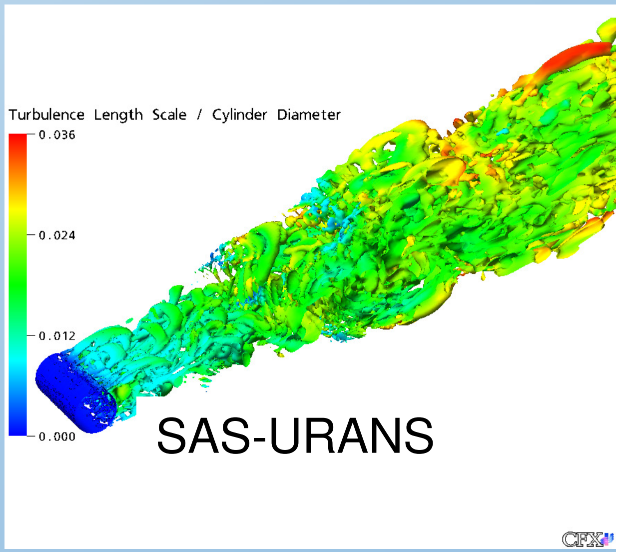
Historic Order of Unsteady Models

- **URANS**
  - URANS gives unphysical single mode unsteady behavior
- **LES (Large Eddy Simulation)**
  - Too expensive for most industrial flows due to high resolution requirements in boundary layers
- **DES (Detached Eddy Simulation)**
  - First industrial-strength model for high-Re with LES-content
  - Increased complexity (grid sensitivity) due to explicit mix of to modelling concepts
- **SAS (Scale-Adaptive Simulation)**
  - Extends URANS to many technical flows
  - Provides “LES”-content in unsteady regions.



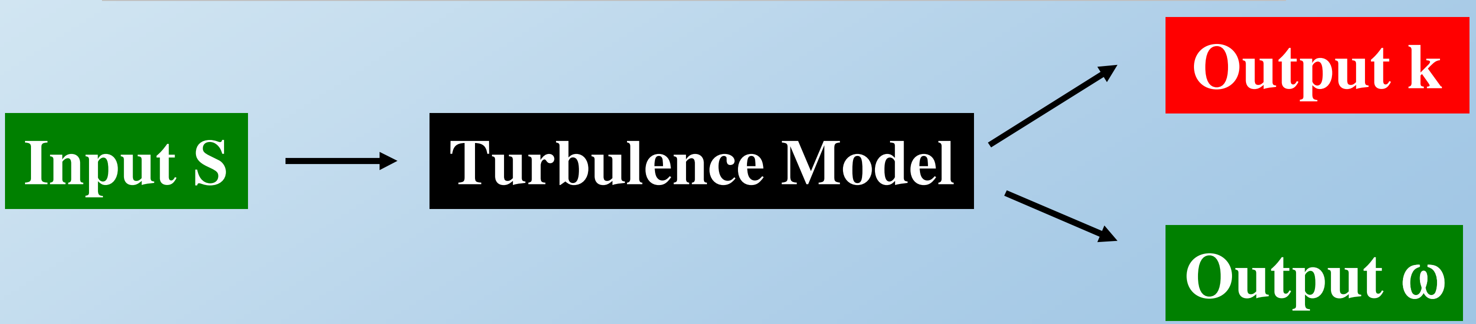


Two-Equation Model Scales

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho U_j k)}{\partial x_j}=\mu_t\left(S^2-c_\mu\omega^2\right)+\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho U_j\omega)}{\partial x_j}=\rho\left(c_{\omega 1}S^2-c_{\omega 2}\omega^2\right)+\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\omega}\frac{\partial\omega}{\partial x_j}\right)$$

One input scale – two output scales?



Contradiction?

Determination of L in k-ω Model

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k-equation:

$$\frac{\partial (k)}{\partial t}+\frac{\partial (U_k k)}{\partial x_k}=\frac{k}{\omega}(S^2-c_\mu\omega^2)+\frac{\partial}{\partial y}\left[\frac{k}{\omega}\frac{\partial k}{\partial y}\right]$$

- **Diffusion term carries information on shear-layer thickness δ**
- **Finite thickness layer required**
- **No scale-resolution inside layer – independent of level of S in shear layer**

$$0=\frac{k}{\omega}(S^2-c_\mu\omega^2)+c\frac{1}{\delta}\left[\frac{k}{\omega}\frac{k}{\delta}\right]$$

$$\omega \sim S$$ **from ω-equation**

$$L_t \sim \frac{\sqrt{k}}{\omega} \sim \frac{\sqrt{S^2\delta^2}}{S} \sim \delta$$

Definition of URANS Model?

Transport Equation Integral Length-Scale (Rotta)

- **Exact transport equations for Φ=*kL* (shear layer):** $kL(x,t) = \int_{-\infty}^{\infty} R_{ii}(x,t,r)dr$

$$\frac{\partial(\Phi)}{\partial t} + \frac{\partial(U_k\Phi)}{\partial x_k} = -\frac{3}{16}\frac{\partial U(x)}{\partial y}\int R_{21}dr_y - \frac{3}{16}\int \frac{\partial U(x+r_y)}{\partial y}R_{12}dr_y +$$

$$\frac{3}{16}\int \frac{\partial}{\partial r_k}\left(R_{(ik)i} - R_{i(ik)}\right)dr_y + \nu\frac{3}{8}\int \frac{\partial^2 R_{ii}}{\partial r_k \partial r_k}dr_y -$$

$$\frac{\partial}{\partial y}\left\{\frac{3}{16}\int\left[R_{(i2)i} + \frac{1}{\rho}\left(\overline{p'v} + \overline{vp'}\right)\right] - \nu\frac{\partial}{\partial y}(\Phi)\right\}$$

- **Important term:**

$$\frac{3}{16}\int \frac{\partial U(x+r_y)}{\partial y}R_{12}dr_y$$

Expansion of Gradient Function

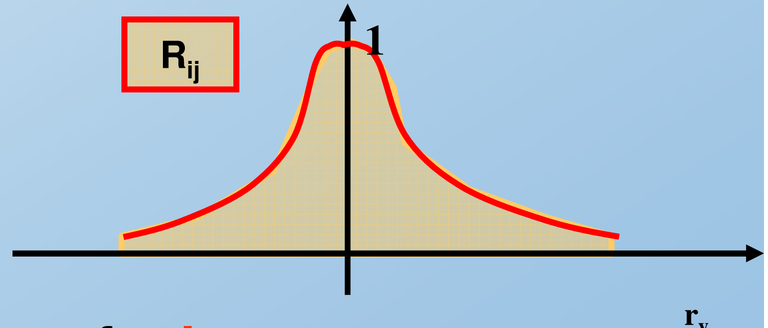
- **Important term:**

$$\frac{\partial U(x+r_y)}{\partial y} = \frac{\partial U(x)}{\partial y} + \frac{\partial^2 U(x)}{\partial y^2} r_y + \frac{\partial^3 U(x)}{\partial y^3}\frac{r_y^2}{2} + \ldots$$

$$\int \frac{\partial U(x+r_y)}{\partial y} R_{12} dr_y \rightarrow \frac{\partial U(x)}{\partial y}\int R_{12} dr_y + \frac{\partial^2 U(x)}{\partial y^2}\int r_y R_{12} dr_y + \frac{1}{2}\frac{\partial^3 U(x)}{\partial y^3}\int r_y^2 R_{12} dr_y$$

- **Rotta:**

$$\frac{\partial^2 U(x)}{\partial y^2}\int r_y R_{12} dr_y = 0$$



- **Due to symmetry of R_{12} with respect to r_y for homogeneous turbulence**

Transport Equation Integral Length-Scale (Rotta)

- **Transport equations for *kL*:**

$$\frac{\partial(\rho\Phi)}{\partial t}+\frac{\partial(\rho U_k\Phi)}{\partial x_k}=-\rho\overline{uv}\left(\zeta L\frac{\partial U_i(x)}{\partial y}+\zeta_3 L^3\frac{\partial^3 U_i(x)}{\partial y^3}\right)-c_L c\rho\left(\frac{q^2}{2}\right)^{3/2}+\frac{\partial}{\partial y}\left\{\frac{\mu_t}{\sigma_\Phi}\frac{\partial}{\partial y}(\Phi)\right\}$$

- **Equation has a natural length scale:**

$$L^2=\frac{c_l-c}{\zeta_3}\left|\frac{\partial U/\partial y}{\partial^3 U/\partial y^3}\right|$$

- **Problem – 3rd derivative:**
  - **Non-intuitive**
  - **Numerically problematic**

→ $\zeta_3=0$ →

- **If ζ_3=0 - No natural length scale**
  - **No fundamental difference to other scale-equations**

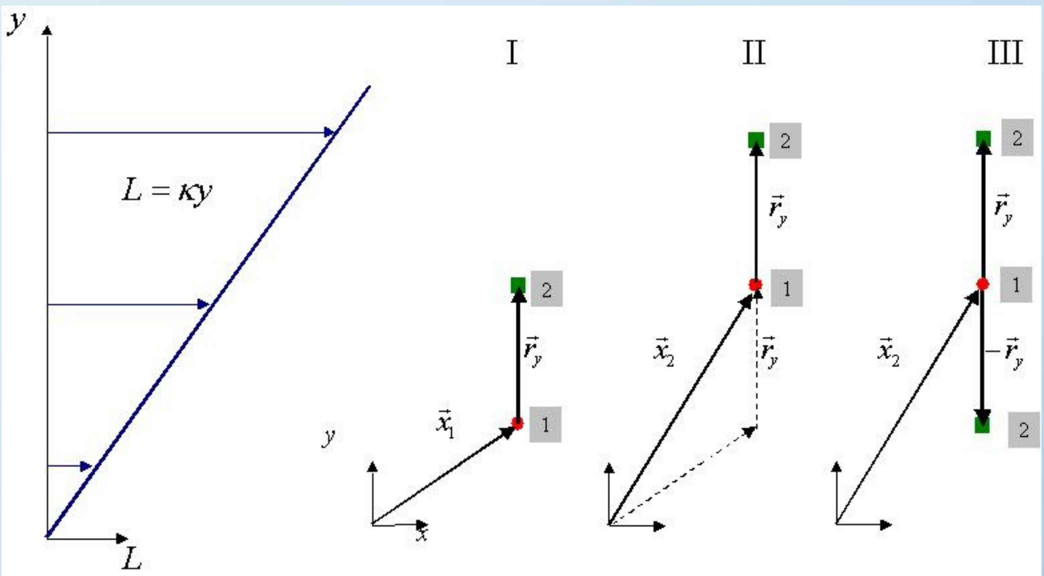
Virtual Experiment 1D Flow

Logarithmic layer $L_t = \kappa y$

$$\int r_y R_{12} dr_y = 0 \quad ?$$

$$\tilde{R}_{12} = \frac{\overline{u(x)v(x+r_y)}}{\overline{u(x)v(x)}}$$

$$\overline{u(x)v(x)} = const. = \frac{\tau_w}{\rho}$$



$$\tilde{R}^{I}_{12}(\vec{r}_y) < \tilde{R}^{II}_{12}(\vec{r}_y)$$

$$\tilde{R}^{III}_{12}(\vec{r}_y) = \tilde{R}^{II}_{12}(\vec{r}_y) \qquad \tilde{R}^{III}_{12}(\vec{r}_y) \approx \tilde{R}^{I}_{12}(-\vec{r}_y)$$

$$\tilde{R}^{III}_{12}(-\vec{r}_y) < \tilde{R}^{III}_{12}(\vec{r}_y)$$

$\longrightarrow$

R_{12} asymmetric

$$\frac{\partial^2 U(x)}{\partial y^2} \int r_y R_{12} dr_y \neq 0$$

New Transport Equation

- **New transport equation for** $\Phi = \sqrt{k}L$

$$\frac{\partial \Phi}{\partial t} + U_j \frac{\partial \Phi}{\partial x_j} = \zeta_1 \frac{\Phi}{k} P_k - \hat{\zeta}_2 \nu_t S \left| U'' \right| \frac{\Phi^2}{k^{3/2}} - \zeta_3 \cdot k + \frac{\partial}{\partial y}\left[\frac{\nu_t}{\sigma_\Phi} \frac{\partial \Phi}{\partial y} \right]$$

$$\left| U'' \right| = \sqrt{\frac{\partial^2 U_i}{\partial x_j \partial x_j} \frac{\partial^2 U_i}{\partial x_k \partial x_k}}; \quad P_k = \nu_t S^2$$

v. Karman length-scale as natural length-scale:

$$L \sim \kappa \left| \frac{S}{U''} \right| = L_{vK}$$

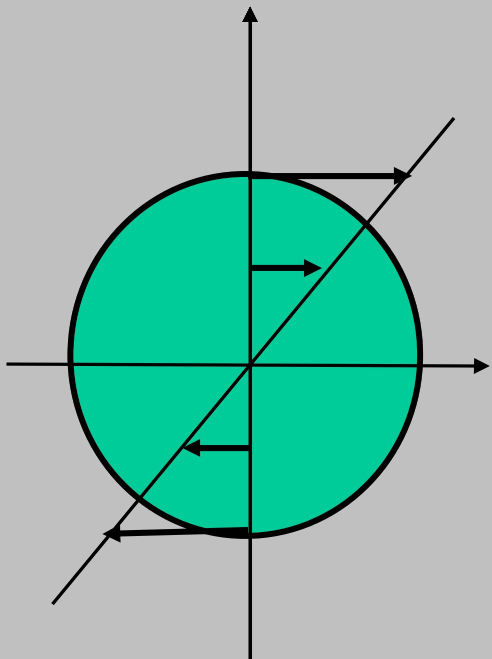
Limitation of Growth by U”

Homogenous Shear

$$\frac{dU}{dy} = const.$$

$$\omega \sim \frac{dU}{dy}$$

$$L \rightarrow \infty$$



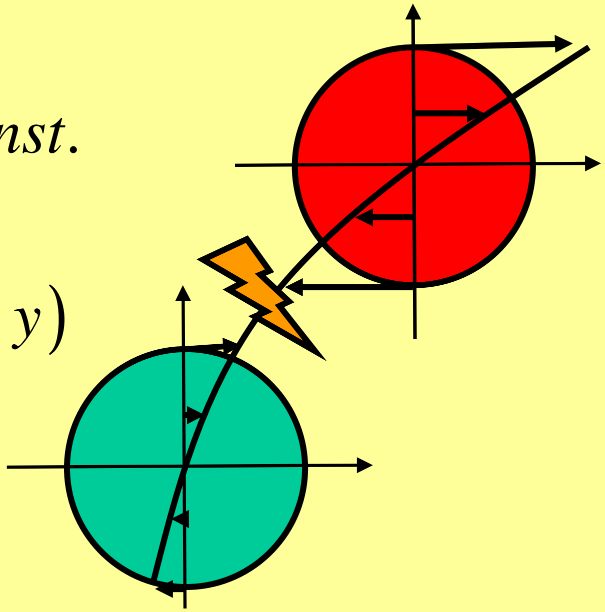
Eddies grow to infinity

Inhomogeneous Shear

$$\frac{dU}{dy} \neq const.$$

$$\omega \sim \frac{dU}{dy}(y)$$

$$L \rightarrow L_{vK}$$



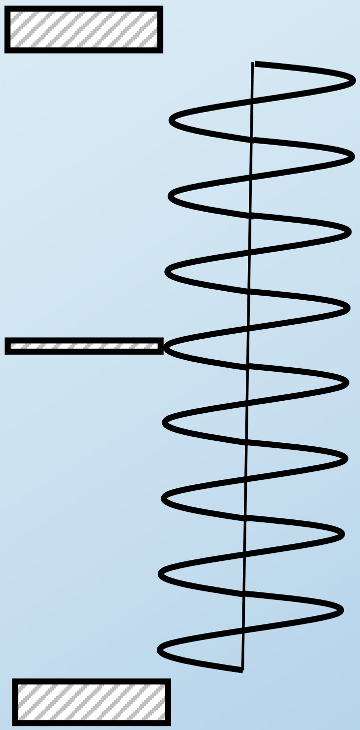
Eddy growth limited by LvK.

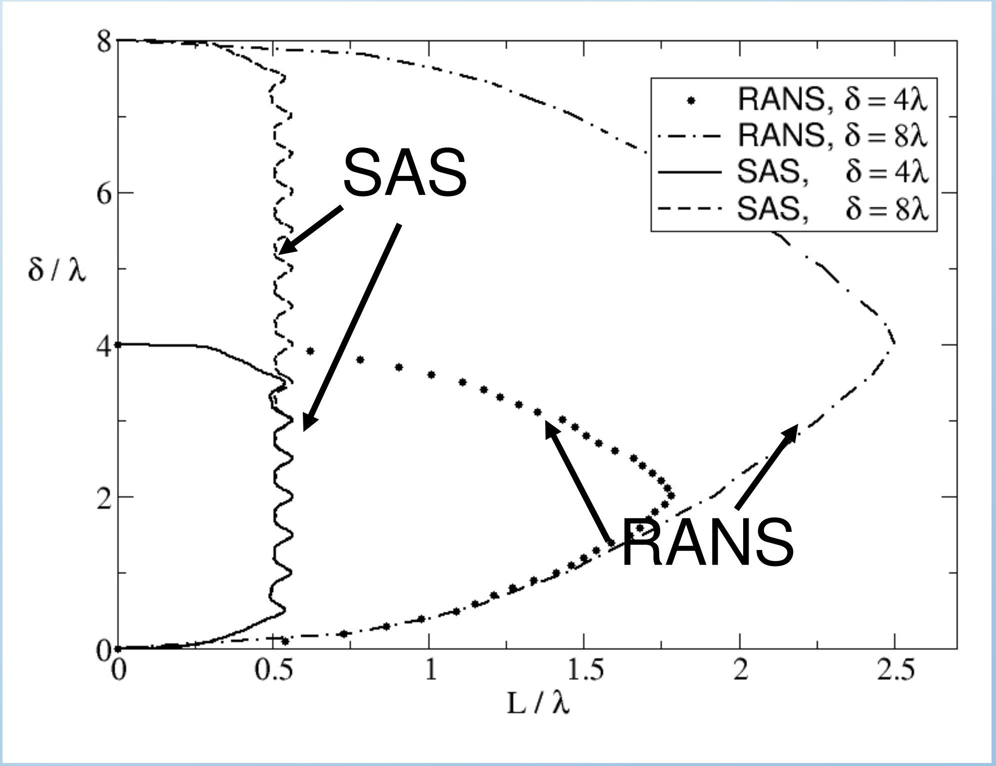
Resolution of Scales

$$U(y) = U_0 \sin\left(\frac{2\pi \cdot y}{\lambda}\right)$$

natural scale

RANS Model L~δ
SAS L~λ





Transformation of SAS Terms to SST Model

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- **Tranformation:** $$\Phi = \frac{1}{c_\mu^{1/4}} \frac{k}{\omega}$$

$$\frac{D\omega}{Dt} = \frac{1}{c_\mu^{1/4}} \frac{D}{Dt}\left(\frac{k}{\Phi}\right) = \frac{1}{c_\mu^{1/4}} \left(\frac{1}{\Phi}\frac{Dk}{Dt} - \frac{k}{\Phi^2}\frac{D\Phi}{Dt}\right) = \frac{\omega}{k}\frac{Dk}{Dt} - \frac{\omega}{\Phi}\frac{D\Phi}{Dt}$$

↓

$$\frac{\partial\rho\omega}{\partial t} + \frac{\partial U_j \rho\omega}{\partial x_j} = \underbrace{\alpha\rho S^2 - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\omega}\frac{\partial\omega}{\partial x_j}\right)}_{\text{Wilcox Model}} + \underbrace{\frac{2\rho}{\sigma_\Phi}\left(\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j} - \frac{k}{\omega^2}\frac{\partial\omega}{\partial x_j}\frac{\partial\omega}{\partial x_j}\right)}_{\text{BSL (SST) Model}} + \underbrace{\tilde{\zeta}_2 \kappa\rho S^2 \frac{L}{L_{\nu K}}}_{\text{New}}$$

Wilcox Model | BSL (SST) Model | New

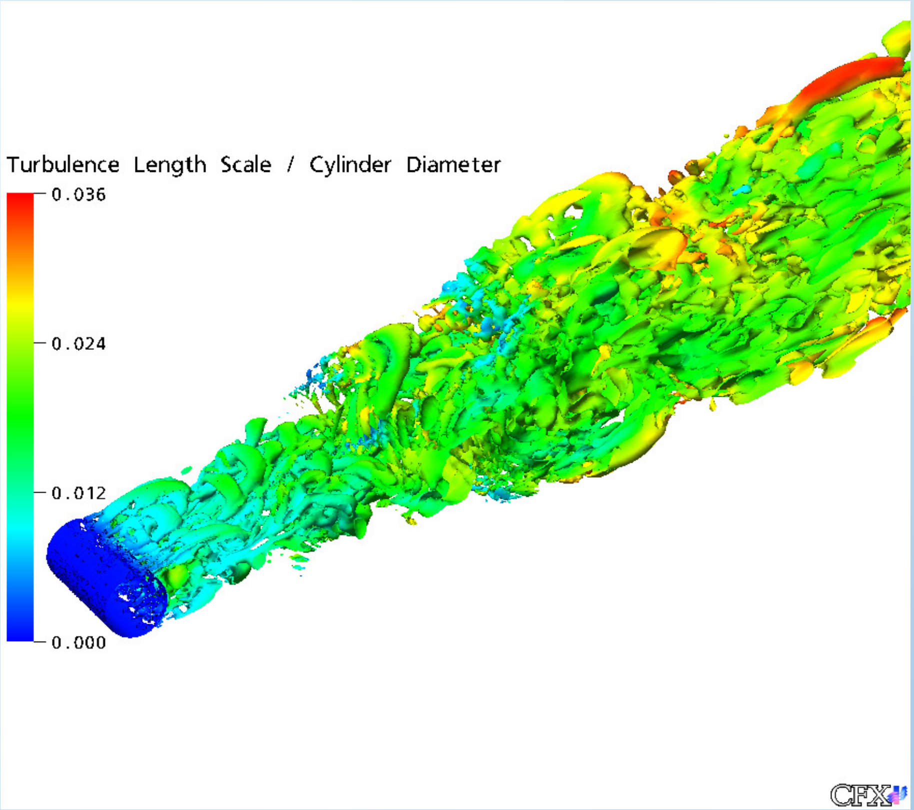
SST-SAS Term →

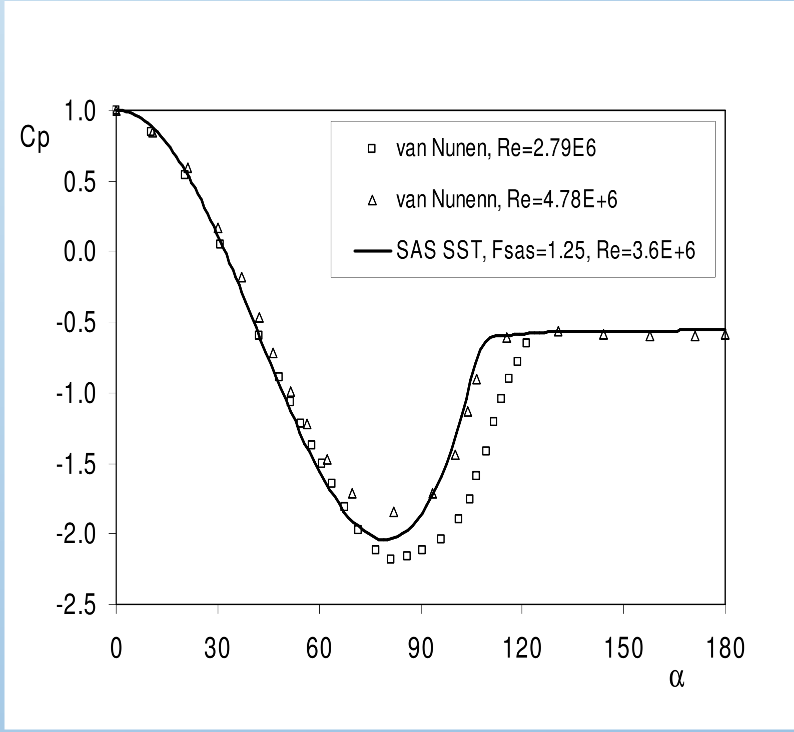
$$F_{SST-SAS} = \rho \cdot F_{SAS} \max\left[\tilde{\zeta}_2 \kappa S^2 \frac{L}{L_{\nu K}} - \frac{2}{\sigma_\Phi} k \cdot \max\left(\frac{1}{\omega^2}\frac{\partial\omega}{\partial x_j}\frac{\partial\omega}{\partial x_j}, \frac{1}{k^2}\frac{\partial k}{\partial x_j}\frac{\partial k}{\partial x_j}\right), 0\right]$$

Cylinder in Cross Flow

- $Re=3.6\times10^6$
- 3.25×10^6 nodes
- $c_{SAS}=0.54$
- Turbulent boundary layer - RANS
- Detached region – "LES"-like

Isosurface of S^2-Ω^2. Colour: *L / D*





SAS Requirements

1. Provide proper solution in the RANS region
2. Initialise large scale instabilities, sustain energy cascade in unsteady regions
3. Provide sufficient energy dissipation of small scales

Hierarchical – Priority 1-3

SAS: Point 3 must be based on information on grid size or the numerical method

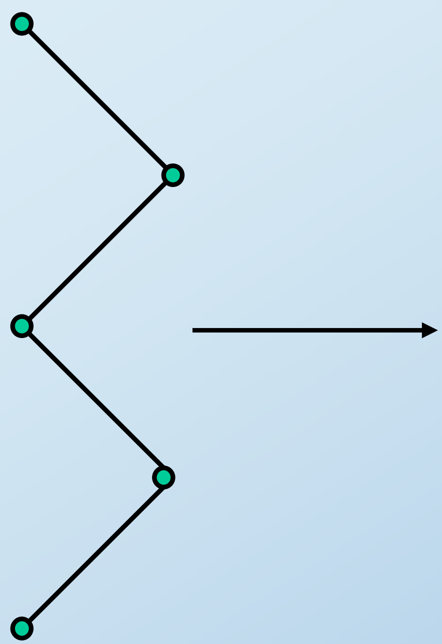
DES: Point 2 and Point 3 are based on grid information

SAS Numerical Requirements

Equilibrium "LES" value of μ_t:

$$\mu_t = \left[\frac{1}{C\cdot\kappa}\left(\frac{\beta}{\beta^*}-\alpha\right)\right]^2\cdot\rho L_{vK}^2 S$$

$U(y)$



$$L_{vK} = \kappa\left|\frac{S}{U''}\right|$$

$$S = \frac{1}{2}\left|\left[\frac{U_{i+1}-U_i}{\Delta}+\frac{U_i-U_{i-1}}{\Delta}\right]\right| = 0 \quad\rightarrow\quad L_{vK}=0$$

$$\overline{S} = \frac{1}{2}\left[\left|\frac{U_{i+1}-U_i}{\Delta}\right|+\left|\frac{U_i-U_{i-1}}{\Delta}\right|\right] \neq 0 \quad\rightarrow\quad L_{vK}=\frac{\Delta}{2}$$

Numerics dependent – L_{vk}=0 – contradiction as resolved scales cannot be smaller than grid spacing

CFX Discretisation

CFX-5 Discretisation – element gradients

- **Production P_k:** $$S^2 = \overline{S^2}$$

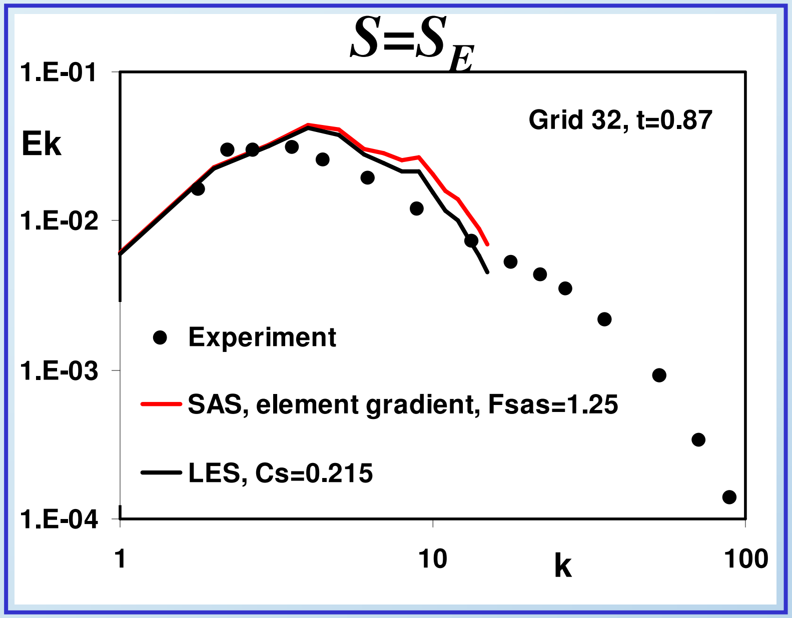
- **SAS-source:** $$\tilde{\zeta}_2 \kappa \overline{S^2} \frac{L}{L_{vK}} \quad , \quad L_{vK} = \frac{\overline{S}}{|U''|}$$

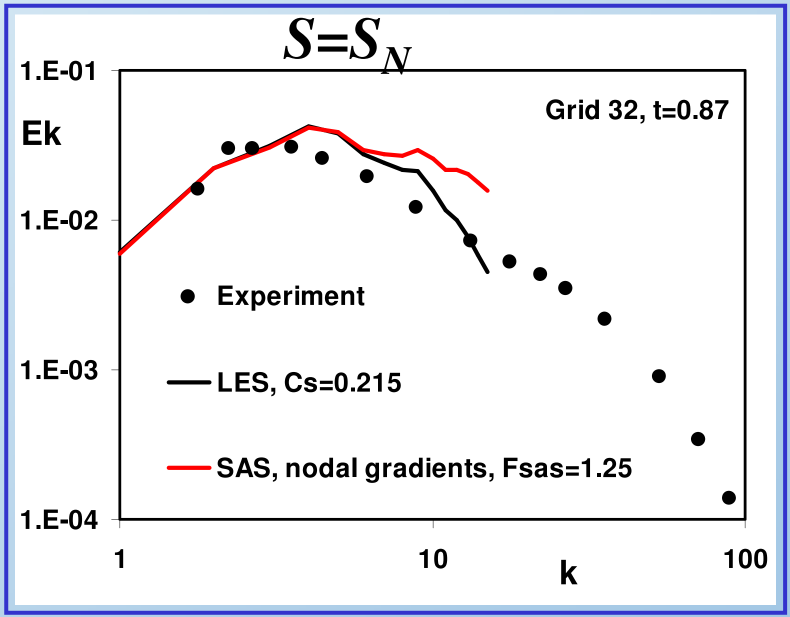
- **"Central difference" gradients for S_{CDS} in P_k and L_{vk} gives much lower damping for unresolved scales as**

$$S_{CDS} < \overline{S}$$

DHIT tests with CFX-5:

- Comparison of nodal (central) and element based discretisation of source terms.



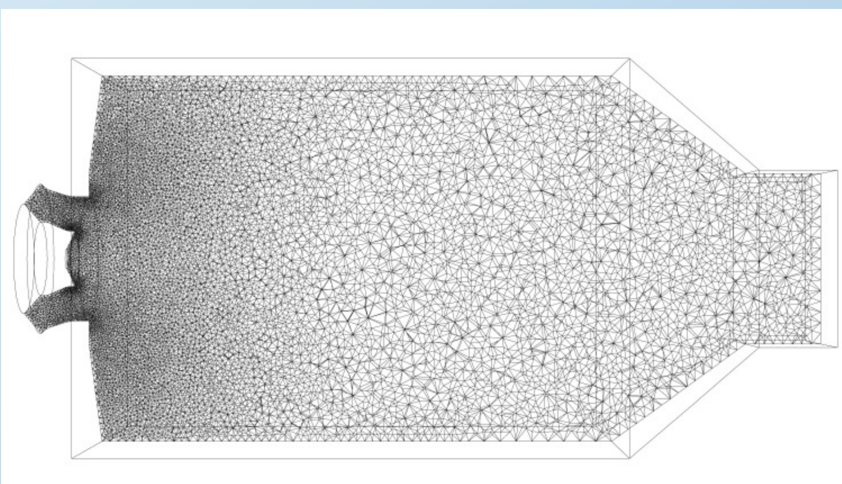


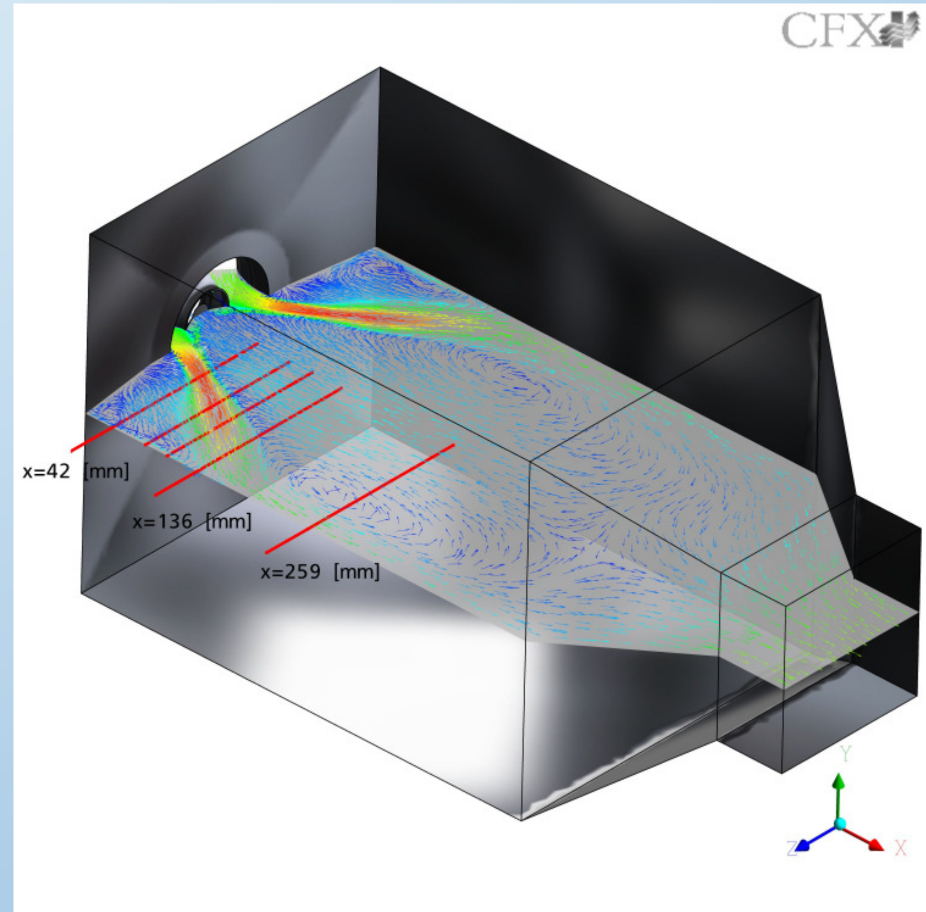
- Alternative is explicit grid information (does not influence RANS solution under grid refinement):

$$\tilde{L}_{vK} = \max(L_{vK}, c\Delta)$$

Gas Combustor (Swirl)

- **Single burner configuration:**
  - Swirl burner of an industrial gas turbine
  - Lean premixed main inlet (methane/air, preheated)
  - Axial dilution air (preheated)
  - Atmospheric pressure
- **Experimental data:**
  - Schildmacher *et al.* [6], [7]
- **600,000 control volumes (3.6×10^6 elements)**





Partially Premixed Combustion Model

- **Reaction progress variable:**
  - Turbulent flamespeed closure [5]
  - Revert to laminar flamespeed for fully resolved limit

$$\frac{\partial(\bar{\rho}\tilde{c})}{\partial t}+\frac{\partial(\bar{\rho}\tilde{u}_j\tilde{c})}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\bar{\rho}D+\frac{\mu_t}{\sigma_c}\right)\cdot\frac{\partial\tilde{c}}{\partial x_j}\right]+\rho_u\, s_T\left|\text{grad}\,\tilde{c}\right|$$

$$s_T = G\max\left(Au'^{3/4}\; s_L^{1/2}\; \lambda_u^{-1/4}\; l_t^{1/4}, s_L\right)$$

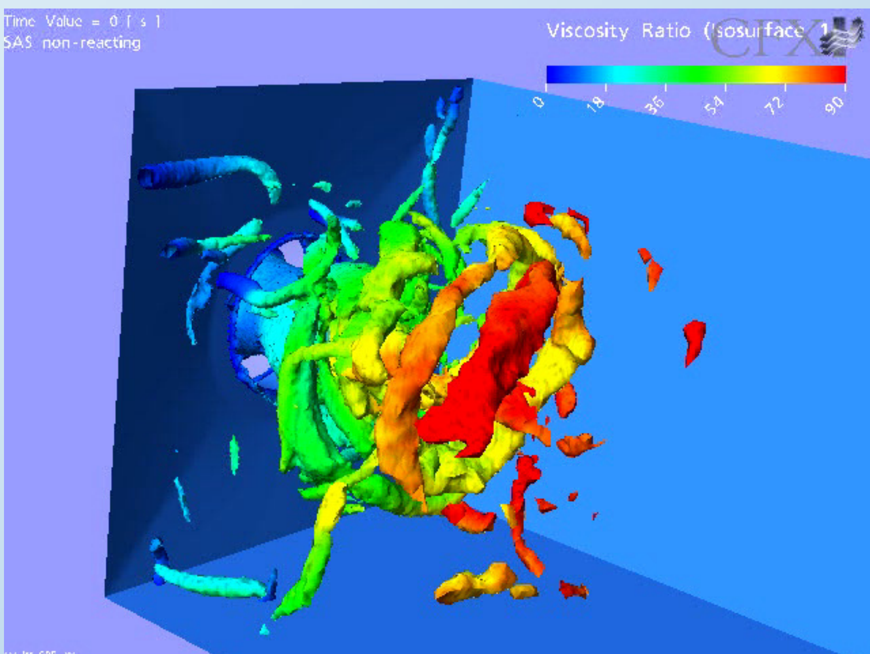
- **Mixture composition:**
  - Blend of fresh and burnt compositions
  - Flamelet model with presumed PDF for burnt fraction

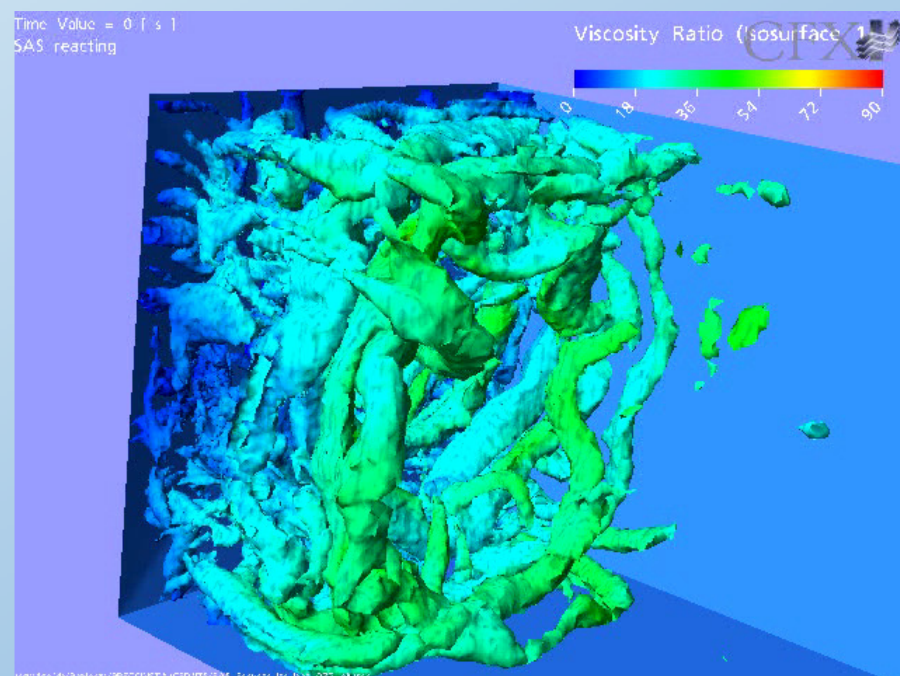
$$\tilde{Y}_\alpha = (1-\tilde{c})\cdot\tilde{Y}_{\alpha,\text{fresh}}\left(\tilde{Z}\right)+\tilde{c}\cdot\tilde{Y}_{\alpha,\text{flamelet}}\left(\tilde{Z},\widetilde{Z''^2},\bar{\chi}\right)$$

Turbulent Structures

- Non-reacting
  - Large characteristic structures near vortex core
  - Higher eddy viscosity to viscosity ratios

- Reacting
  - Turbulent structures more broadly distributed
  - Lower maximum eddy viscosity





Turbulent Structures

- **Non-reacting**
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- **Reacting**
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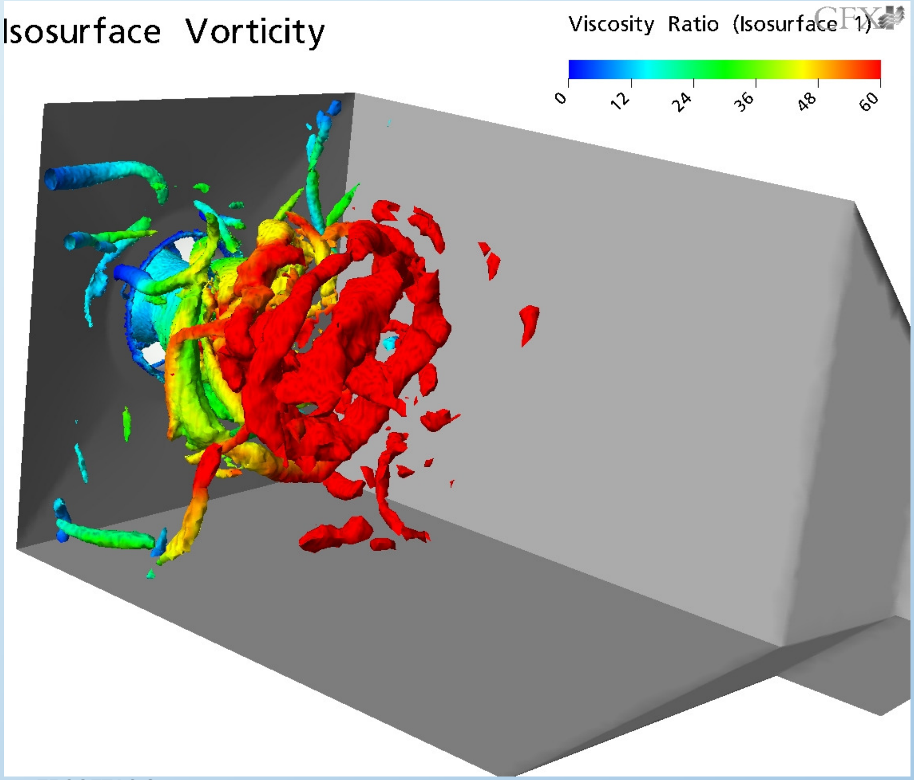
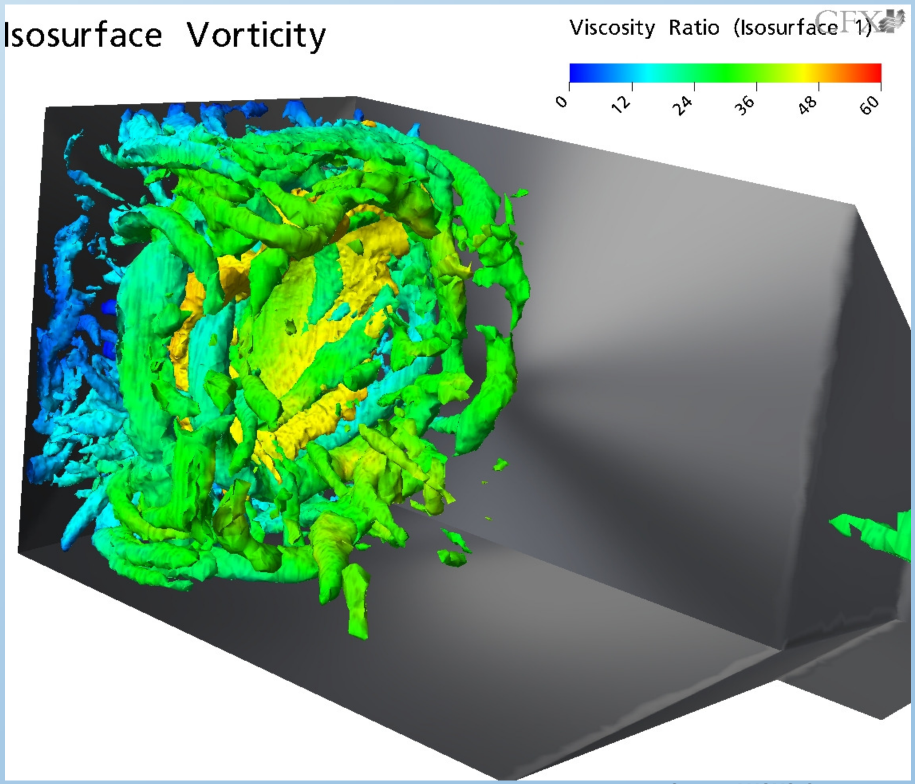
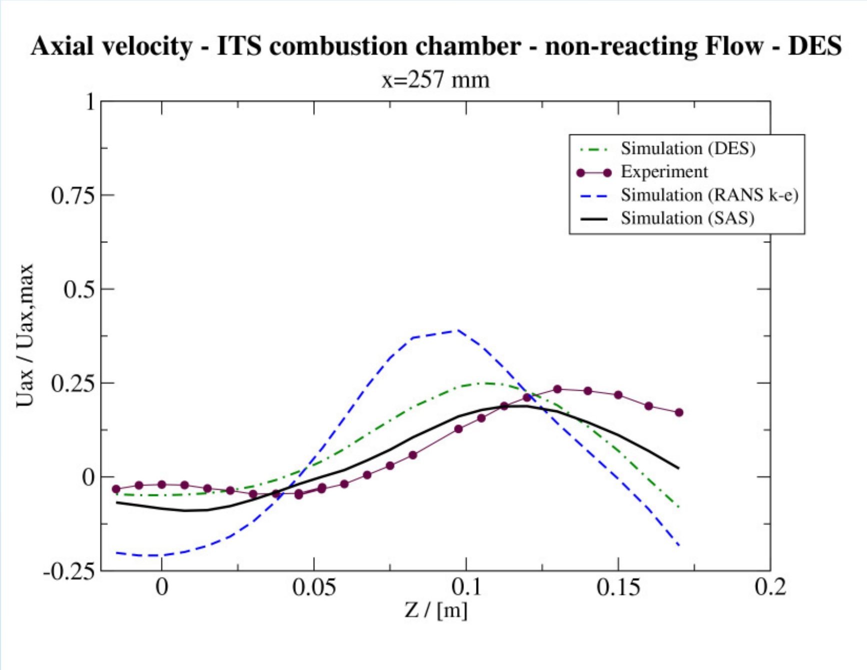
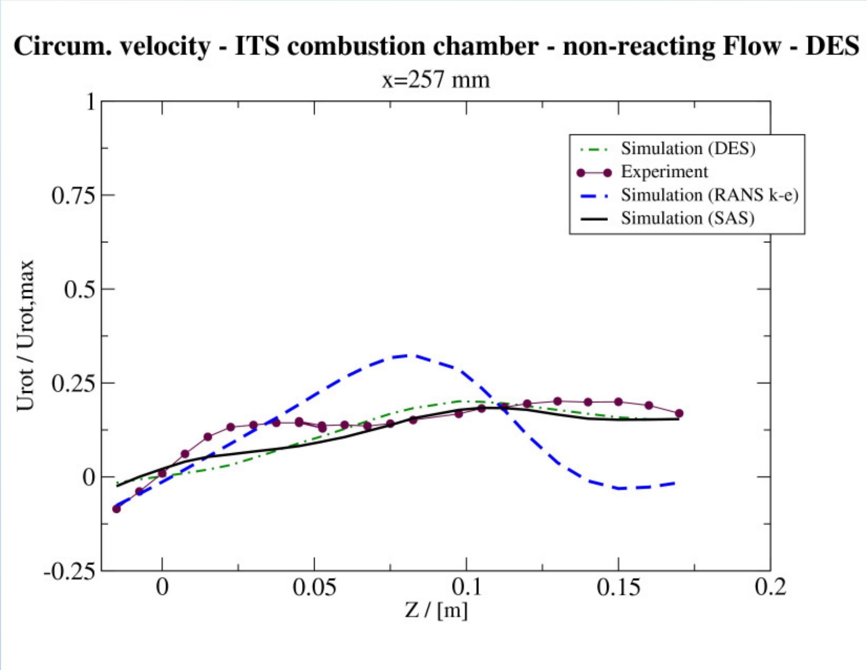
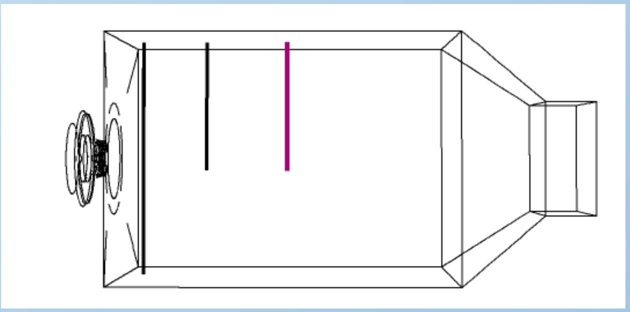




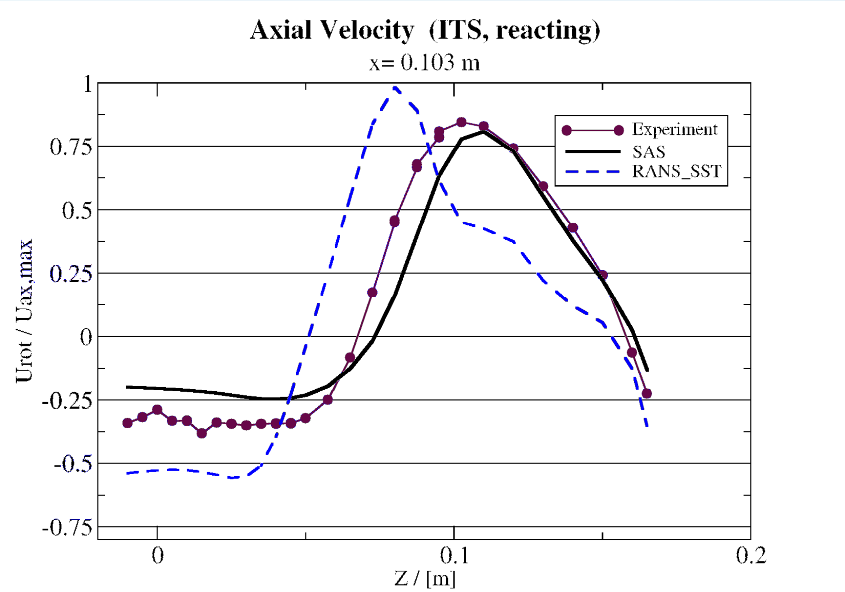
Flow Field: Non-Reacting Flow

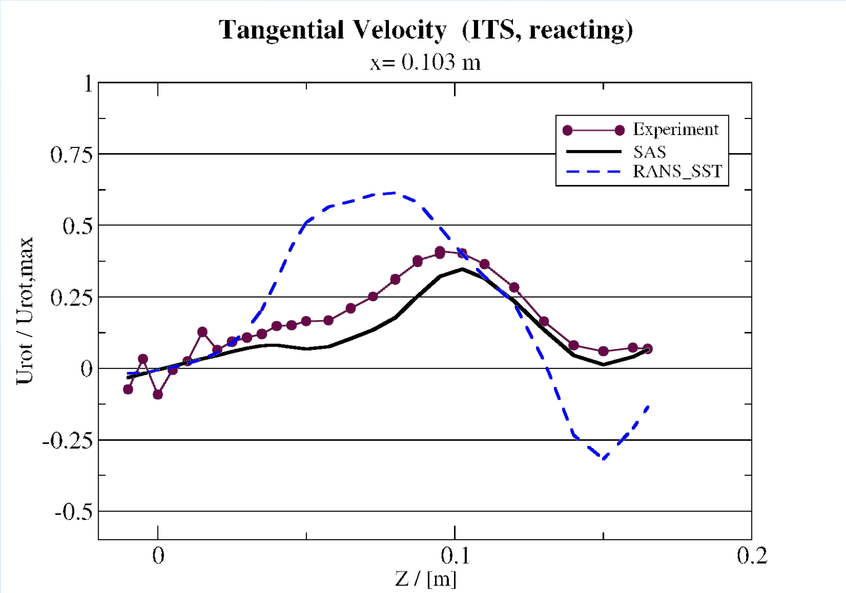






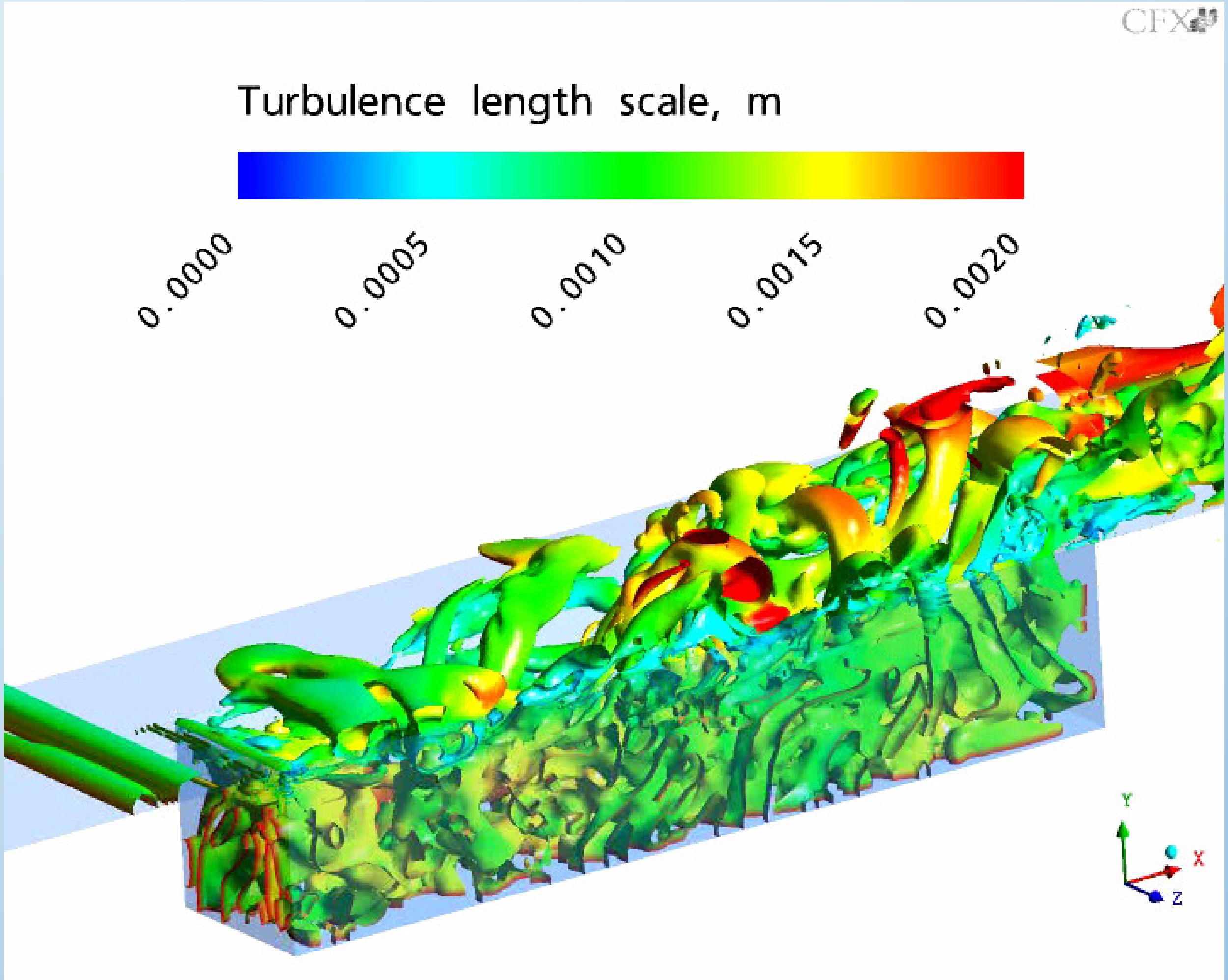
Flow Field: Reacting Flow





Model validation: Cavity

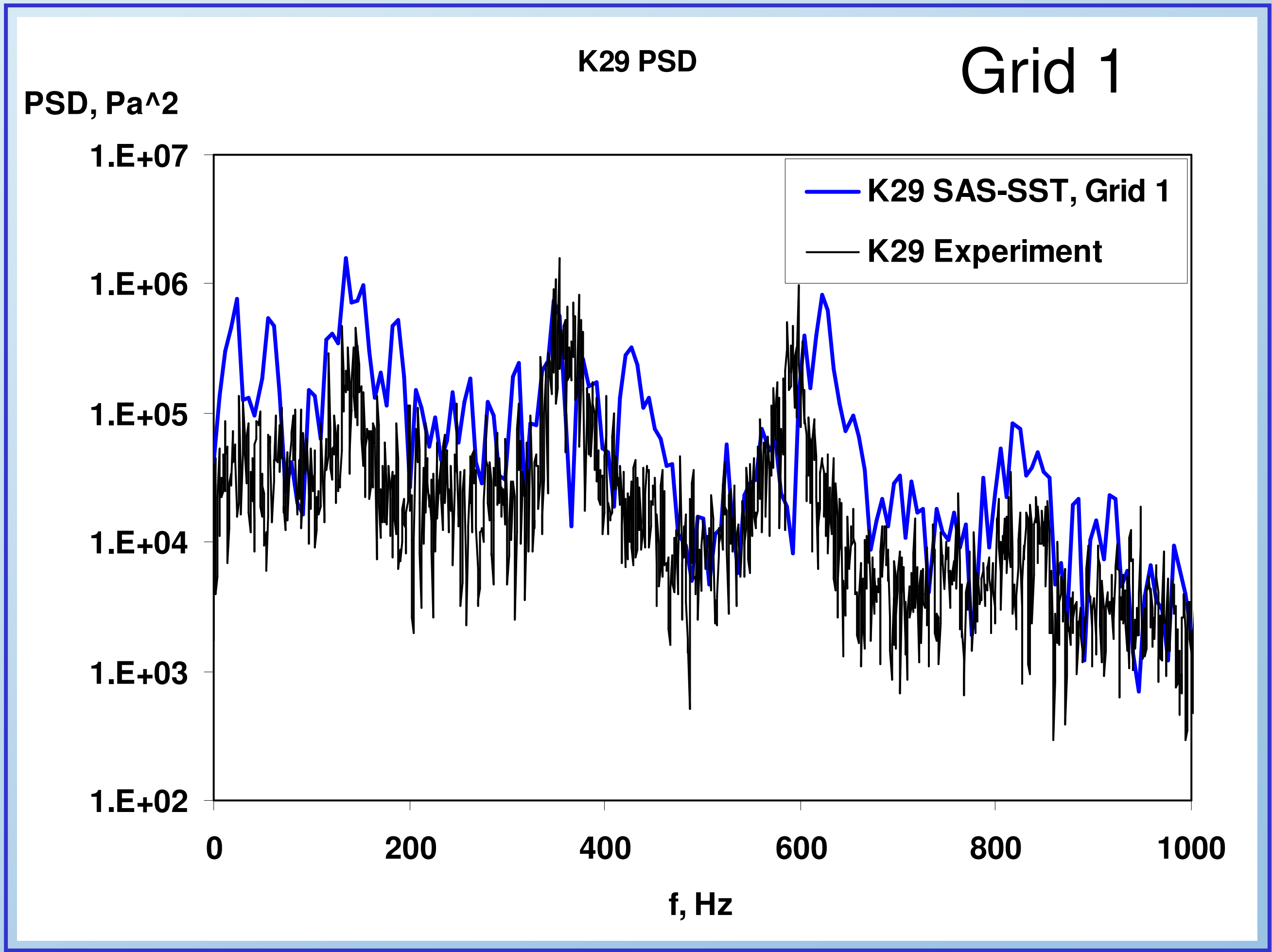
- **Henshaw (2000) M219 case (L×W×D = 5×1×1)**
- **Isosurface $\Omega^2 - S^2 = 10^5\ s^{-2}$**
- **Pressure at K29 probe**
- **~1 million nodes**



Model validation: Cavity

Power spectral density at K29, acoustic range

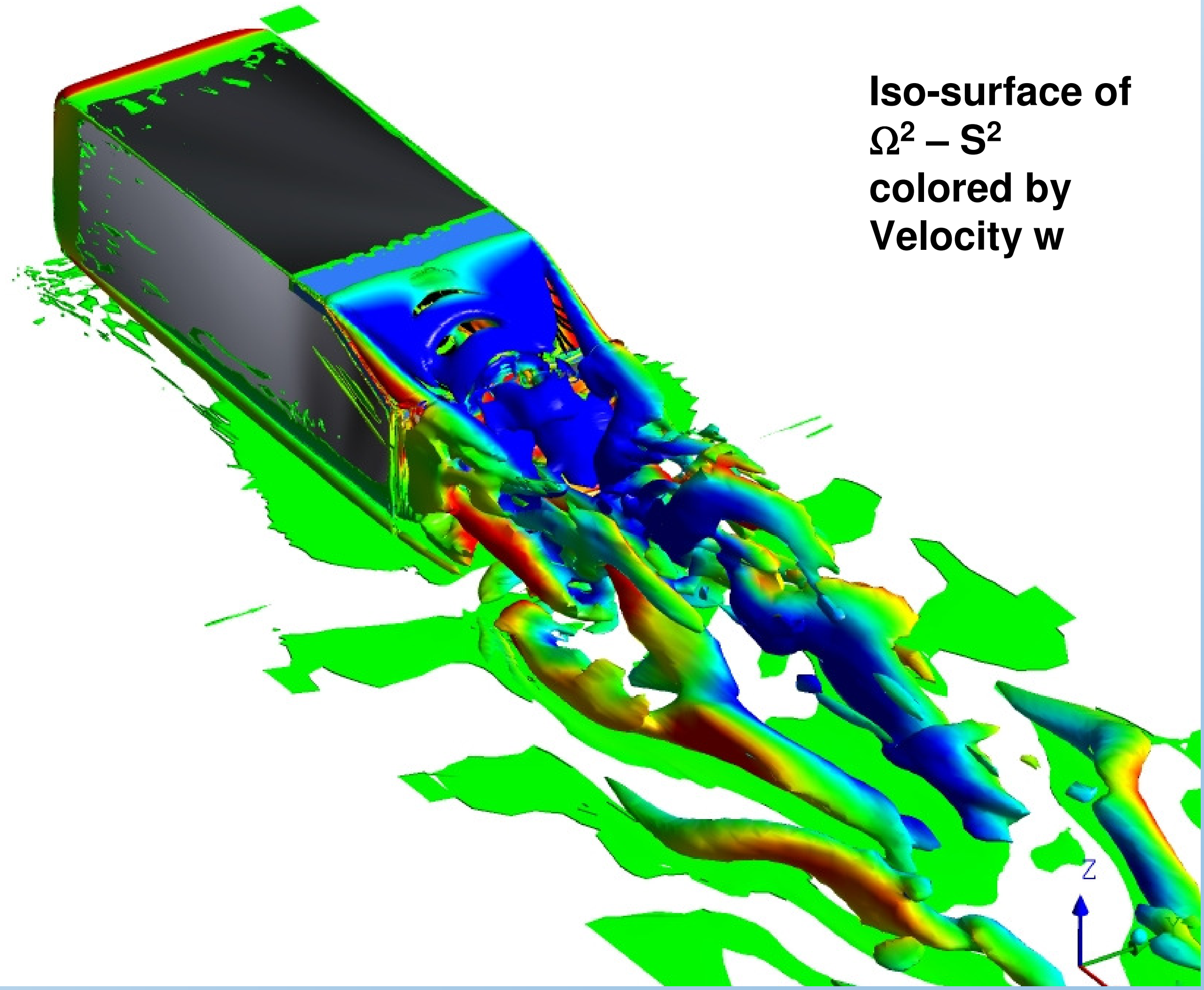
- 2 times less CFD sampling points, than in the experiment
- Currently is running on grid 2



SAS for Ahmed Car, 25° deg slant angle:

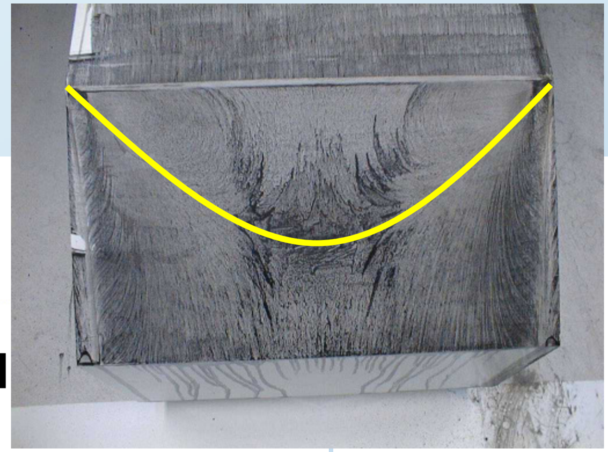
Structured grid with local refinement

~2x10^6 nodes

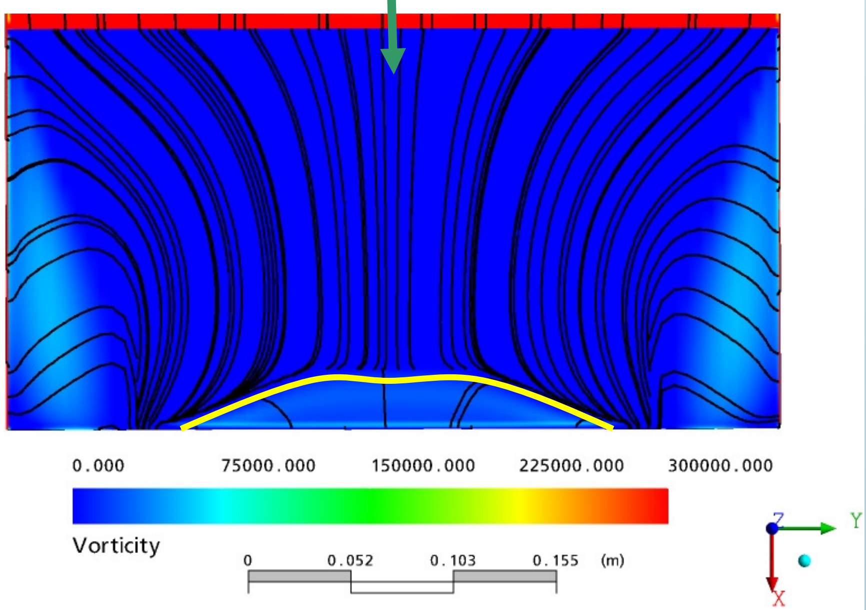


Iso-surface of $\Omega^2 - S^2$ colored by Velocity w

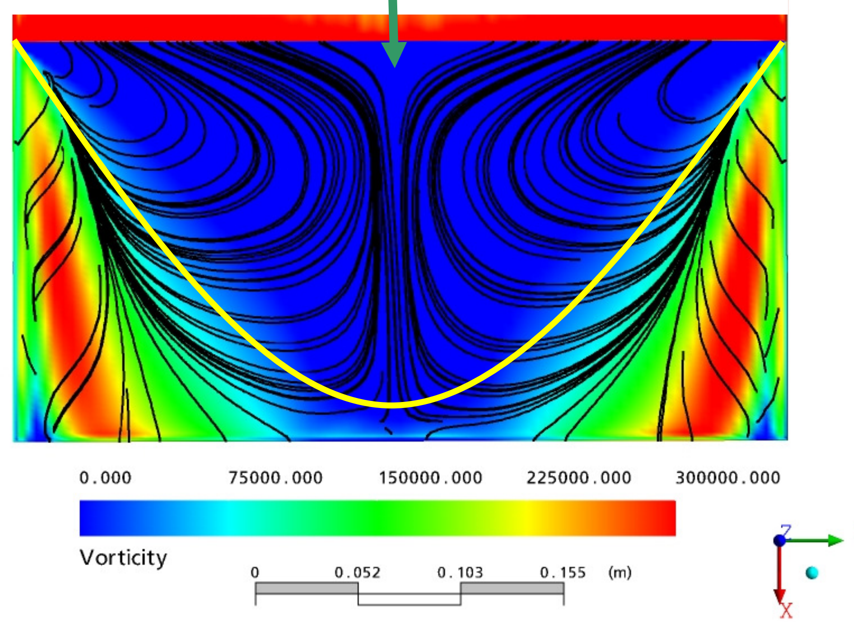
Ahmed Car, 25° deg slant angle: Surface Streamlines and Vorticity



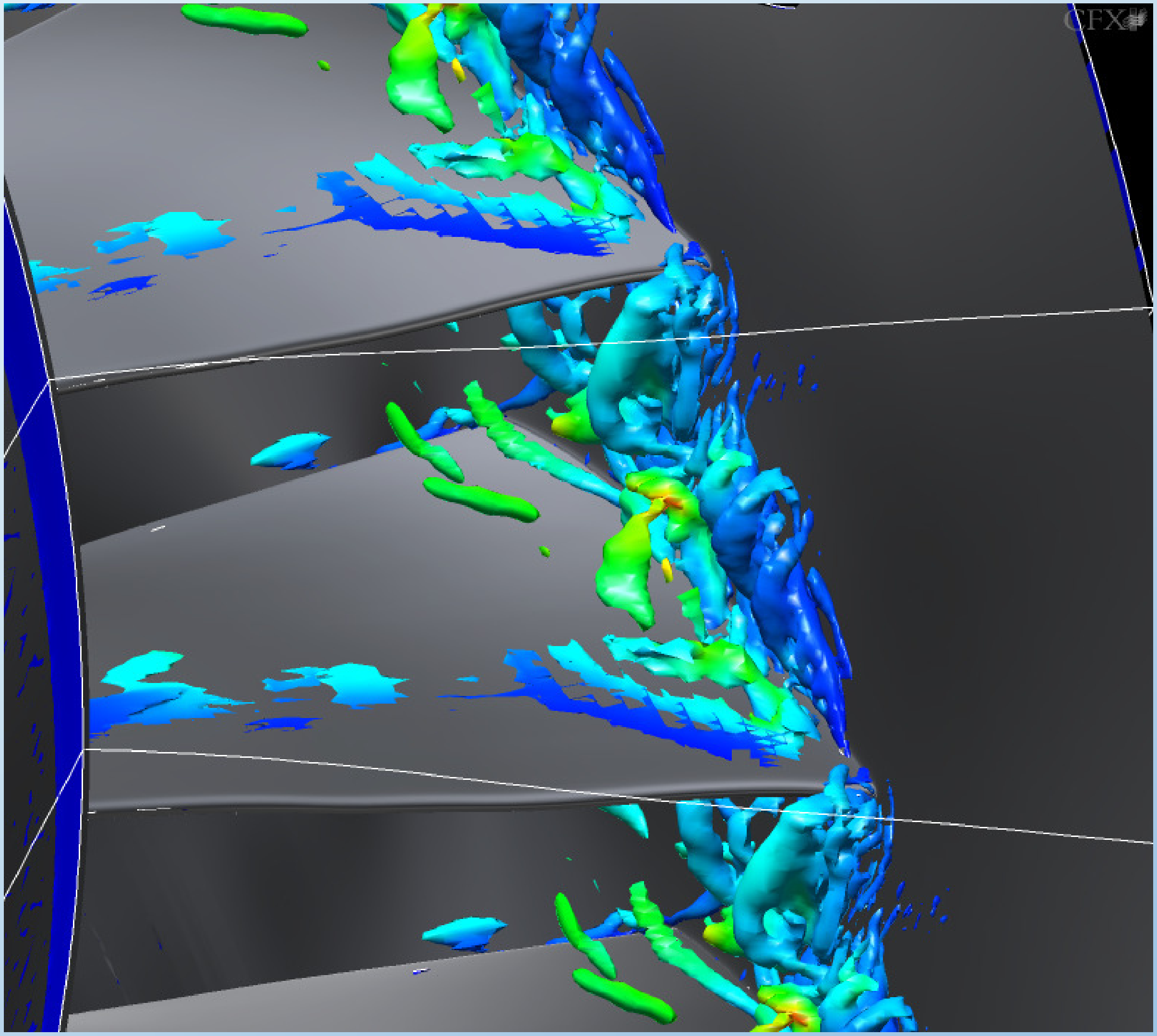
SST, Lift Force = 9.02 N



SAS, Lift Force = 20.77 N



Rotor 35: Turbulent Structures in the tip vortex predicted by SAS



Conclusions

- Standard two-equation models provide <u>single</u> scale through source terms; Length scale proportional to shear layer thickness
- Investigation of exact scale-equation shows that second derivative of velocity field enters into the equation system
- Current knowledge with new model demonstrates proper numerical behavior and numerical results for RANS and SAS mode
- SAS-URANS can do many flows previously only accessible to DES or LES
- Opens question concerning proper definition of URANS