Verfahren hoher Ordnung in der numerischen Aeroakustik und Strömungsmechanik

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Situation for Ordinary Differential Equations

$$\frac{d}{dt}u = f(t,u), \qquad u(0) = u_0$$

Euler-Cauchy method

$$\frac{u^{n+1} - u^n}{\Delta t} = f(t_n, u^n), \qquad u^0 = u_0$$

1st order accurate

$$\text{error} \approx \Delta t$$

Heun method, Crank-Nicolson,

....

2nd order accurate

$$\text{error} \approx \Delta t^2$$

Not used because of efficiency!!

Runge-Kutta, multi-step, extrapolation, ...

Order of accuracy: 4th or higher, or arbitrary

Contents

1. High Order Schemes for CFD
2. ADER-Approach: Arbitrary Order Schemes for CFD and CAA
3. Hybrid Meshes
4. Conclusions

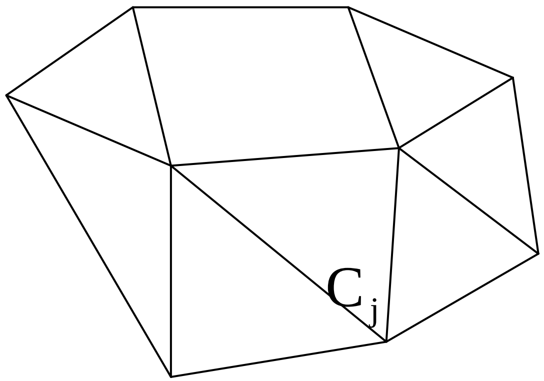
1. High Order Schemes for CFD

- Finite difference schemes (FD-schemes)
  no unstructured grids, no complex geometries
- Finite element schemes (FE-schemes)
  problems at strong gradients
- Finite volume schemes (FV-schemes)
  complex geometries, strong gradients
- Others

Finite-Volume-Schemes

Shock-capturing schemes with

- no spurious oscillations
- second order accuracy



Finite volume schemes

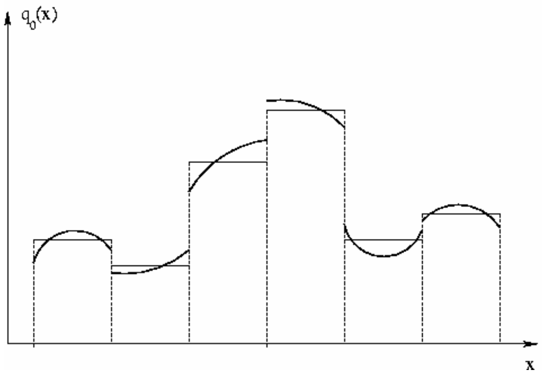
$$u_j^{n+1} = u_j^n - \frac{1}{|C_j|} \int_{t_n}^{t_{n+1}} \int_{\partial C_j} f\left(u(x,t)\right) \cdot \vec{n} \, dS \, dt$$

Evolution equation for integral mean values

Finite Volume Schemes

1. Reconstruction Step

Calculate local values at the grid cell interfaces



Piecewise polynomial reconstruction

2. Flux Calculation between Grid Cells

Nonlinear wave propagation, break-up of discontinuities – Godunov-type, upwind

Standard CFD-Method: MUSCL- Scheme

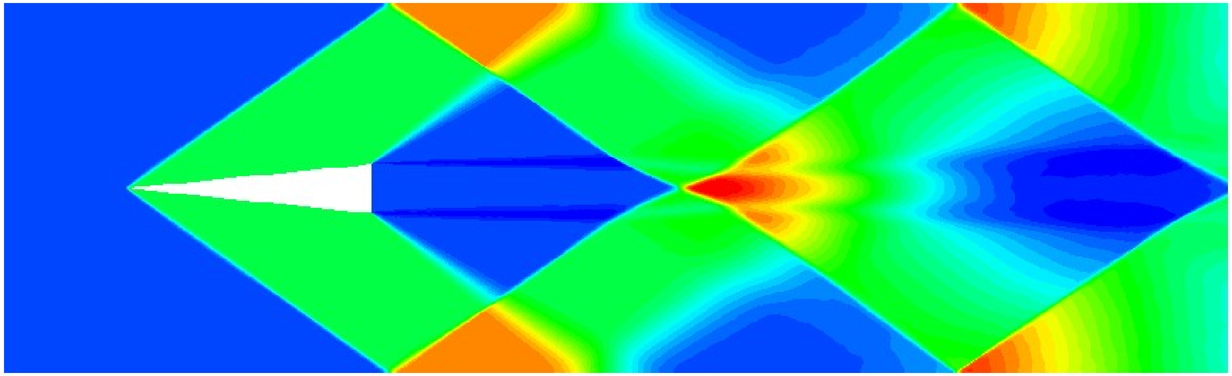
1. Piecewise linear reconstruction

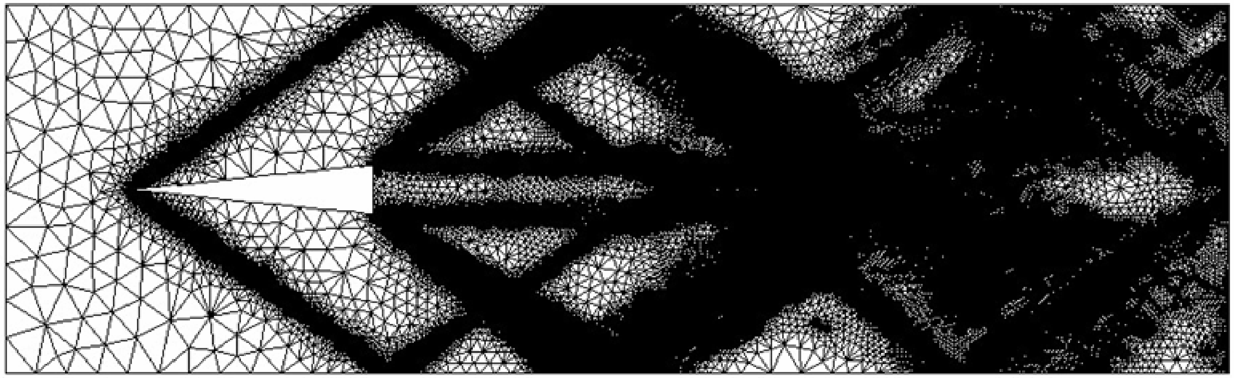
   Second order accuracy in space

2. Time update at $t_{n+1/2}$

   Second order accuracy in time

3. Godunov-type flux calculation





Supersonic flow in a channel over a wedge with h-refinement

Higher Order Accuracy?

Applications: Wave propagation over long distances, turbulent flow, boundary layers, high accuracy,...

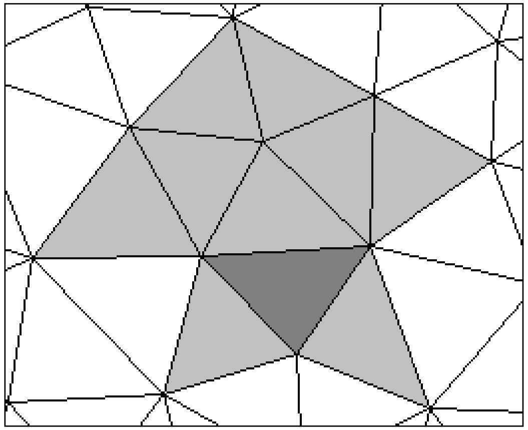
Reconstruction works on structured grids

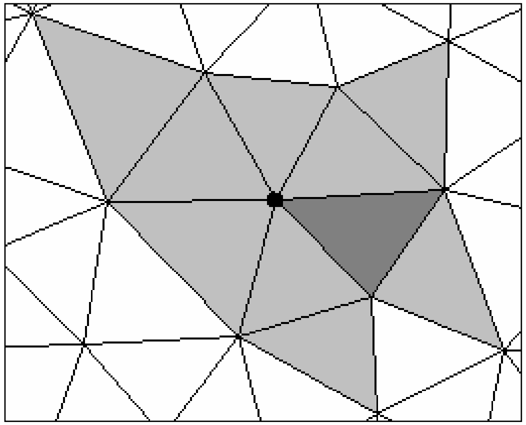
Polynomial reconstruction, ENO, WENO

Problem: Unstructured grids – complex geometries

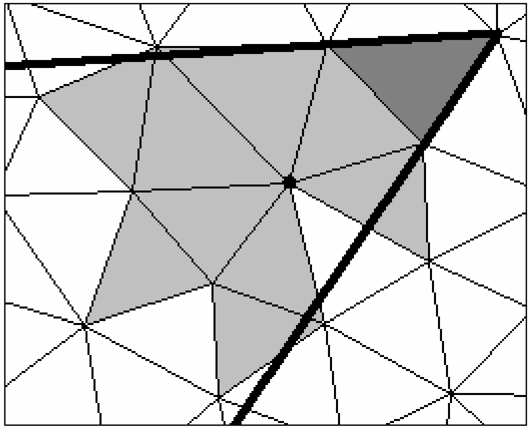
Fourth Order Stencils

- 10 grid cells
- 3 types
  - 3 side face oriented





  - 3 knot oriented



  - 3 sector stencils

Approximation in Time

Runge-Kutta schemes

up to 4-th order of accuracy, (Butcher barrier)

reconstruction at every RK-level !

our experience: (2-dim)

order	effort
1	1
2	2
3	8
4	16

no implementation in 3 dimensions

ADER-approach, T. Toro 1996, Toro+IAG since 1998

Contents

ADER-Approach

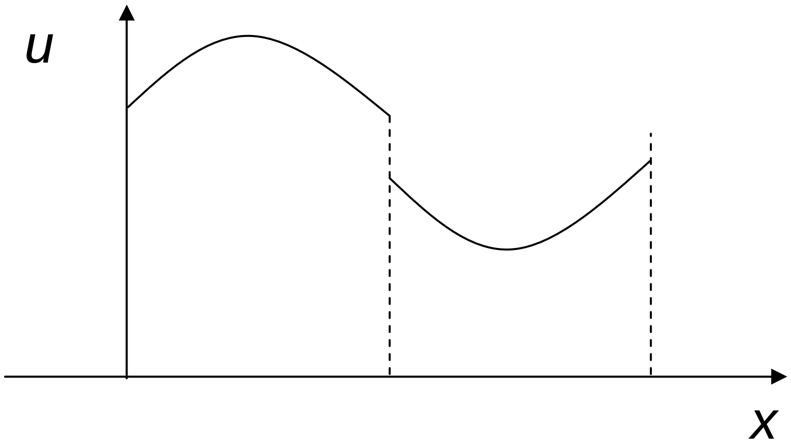
$$u_j^{n+1} = u_j^n - \frac{1}{|C_j|} \int_{t_n}^{t_{n+1}} \int_{\partial C_j} f\left(u\left(x,t\right)\right) \cdot \vec{n} \, dS \, dt$$

Scheme is m-th order accurate, if u is (m-1)-th order accurate

The main idea

1. One Reconstruction
2. Calculate u with high order of accuracy in space and time

Basic Building Block: GRP



$$u_t + f(u)_x = 0$$

$$u(x,0) = \begin{cases} P_i^n(x) & , x < 0 \\ P_{i+1}^n(x) & , x > 0 \end{cases}$$

Generalized Riemann Problem (GRP)

Taylor expansion in time at grid cell interfaces

$$u_B(\tau) = u_B(0) + \sum_{k=1}^{m-1} \frac{\tau^k}{k!} \partial_t^{(k)} u_B(0) + O(\tau^m)$$

Linear Equations on Structured Meshes

Applications: Aeroacoustics, electrodynamics, ...

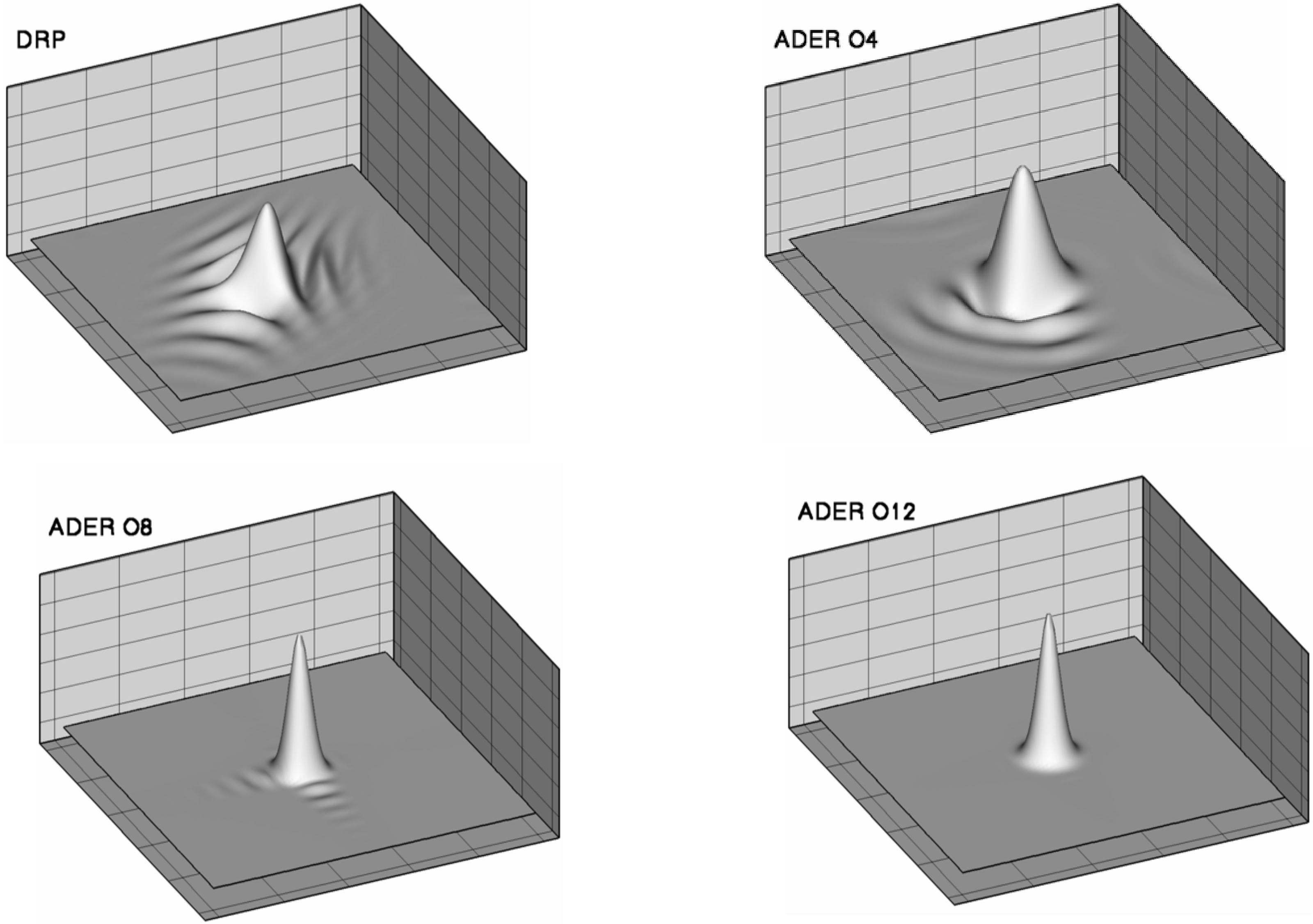
Simplest case: constant coefficients, equidistant grid

$$u_t + Au_x + Bu_y + Cu_z = q$$

hyperbolic system A, B, C matrices

Reformulation with a structure that is similar to a finite difference scheme

Linear advection



ADER – Finite Volume Schemes

1. One reconstruction
2. Generalized Riemann problem

Properties for m-th order accurate

1 reconstruction step, values of functions as well as all derivatives are used

Flux calculation for GRP

One-step scheme: Accuracy in space and time!

less computer time, less memory

Open problem: Unstructured grids

ADER-Discontinuous Galerkin Schemes

Advantage: No reconstruction is necessary

Representation of state vector u in terms of basis functions

$$u_p(x,t) = \hat{u}_{p0}(t)\Phi_0(x) + \hat{u}_{p1}(t)\Phi_1(x) + \ldots + \hat{u}_{pN_d}(t)\Phi_{N_d}(x)$$

$$= \hat{u}_{pl}(t)\Phi_l(x)$$ index summation

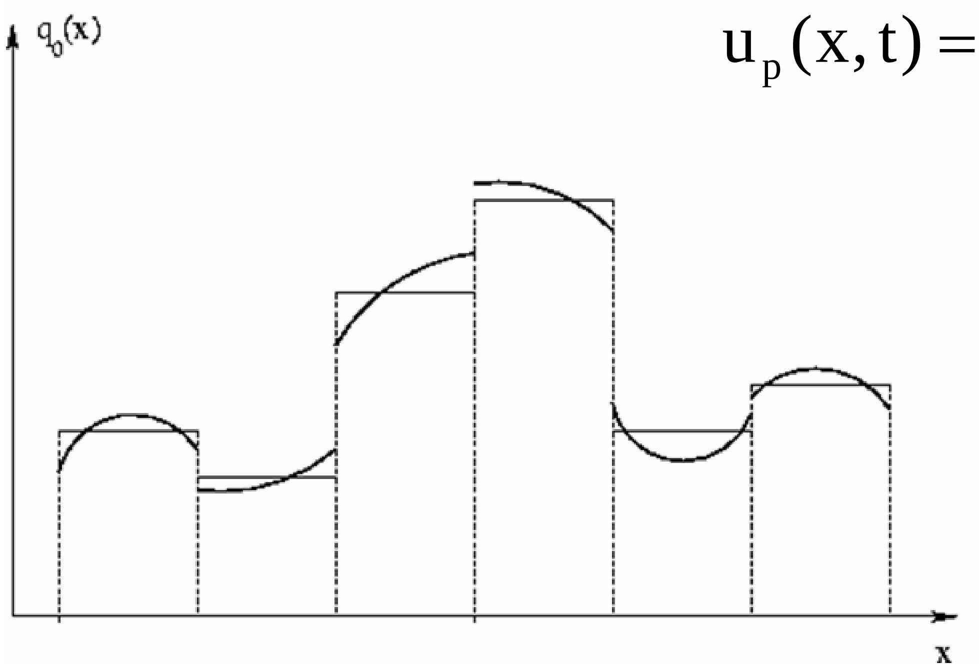
Multiplication with test functions Φ_k and integration by parts

$$\iiint_\Omega \Phi_k \cdot u_t \, dV + \iiint_\Omega \Phi_k \cdot \mathrm{div}(F)\, dV = 0$$

$$\iiint_\Omega \Phi_k \cdot u_t \, dV + \oiint_{\partial\Omega} \Phi_k F \cdot n \, dS - \iiint_\Omega F \mathrm{grad}(\Phi_k)\, dV = 0$$

The data are already stored as piece-wise polynomials

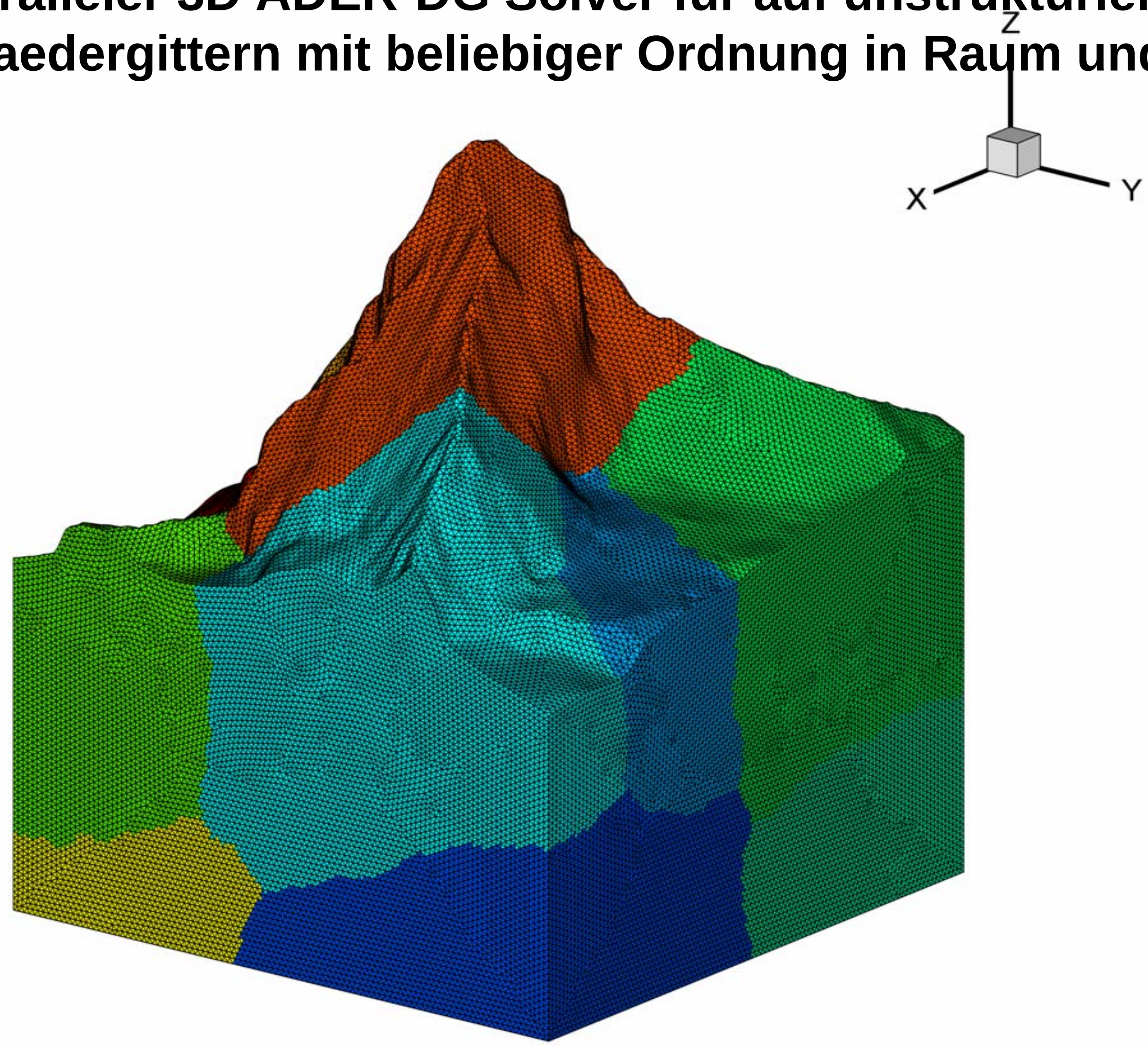
$$u_p(x,t) = \sum_{l=0}^{N_p} \hat{u}_{pl}(t) \cdot \Phi_l(x)$$

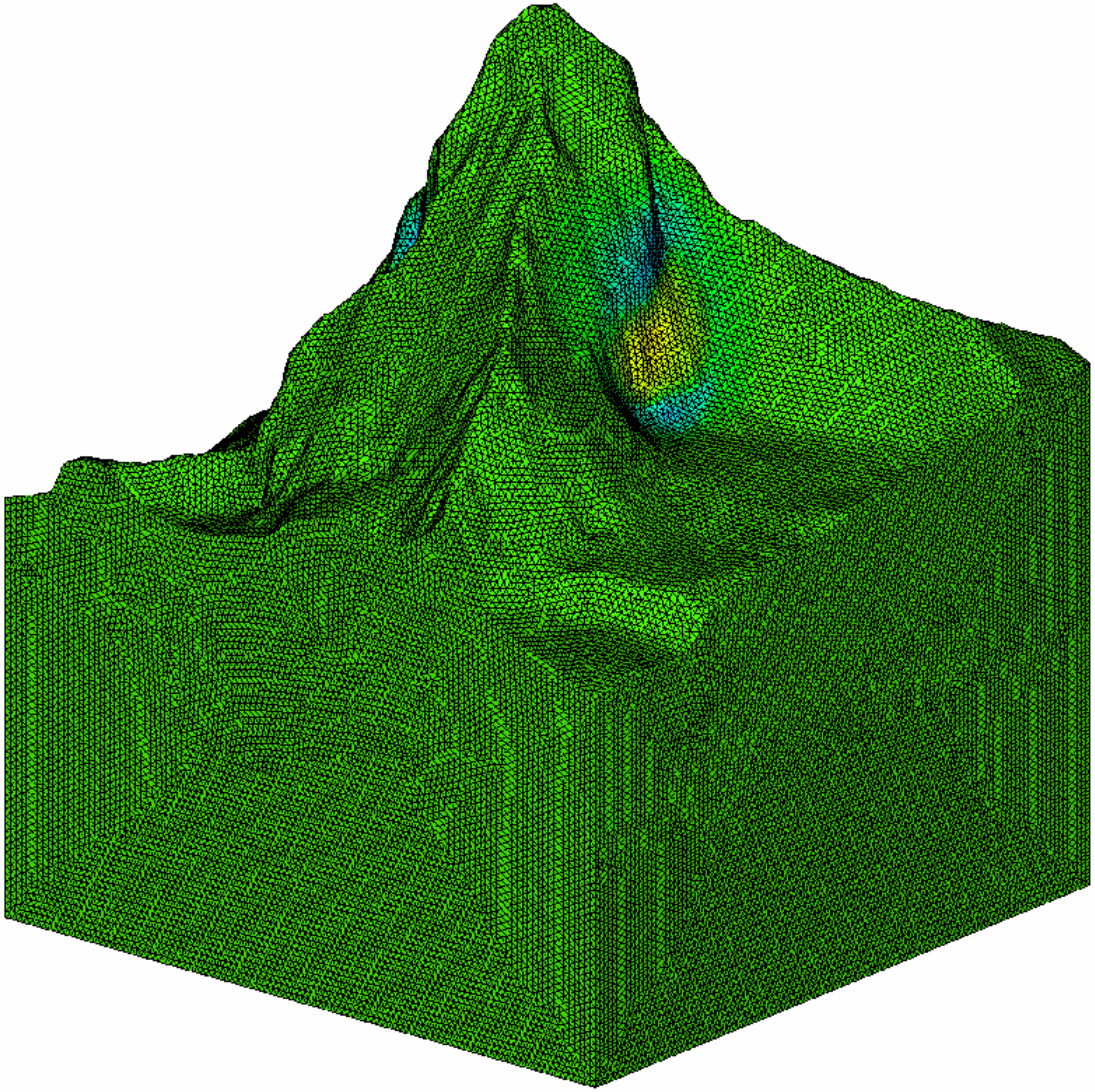


Properties of the ADER-DG Schemes

- Arbitrary high order of accuracy in space and time, even in complex geometries
- Explicit single-step scheme
- Less memory than a classical Runge-Kutta time approximation
- Problems:
  - Large computational effort
  - Viscous terms

Paralleler 3D ADER-DG Solver für auf unstrukturierten Tetraedergittern mit beliebiger Ordnung in Raum und Zeit



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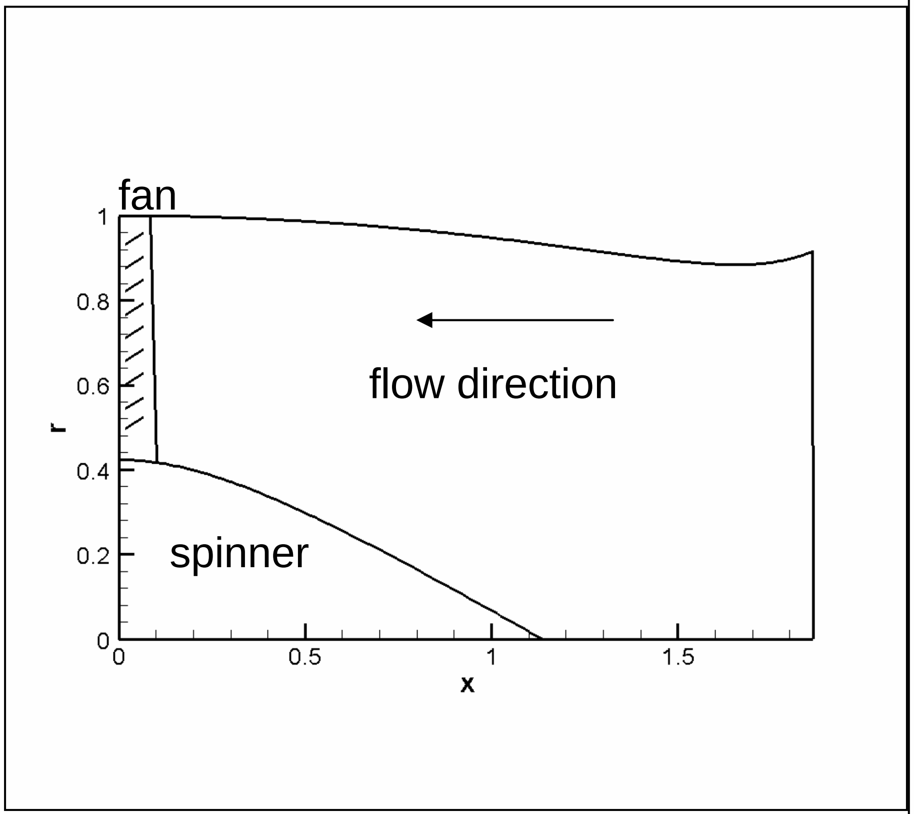
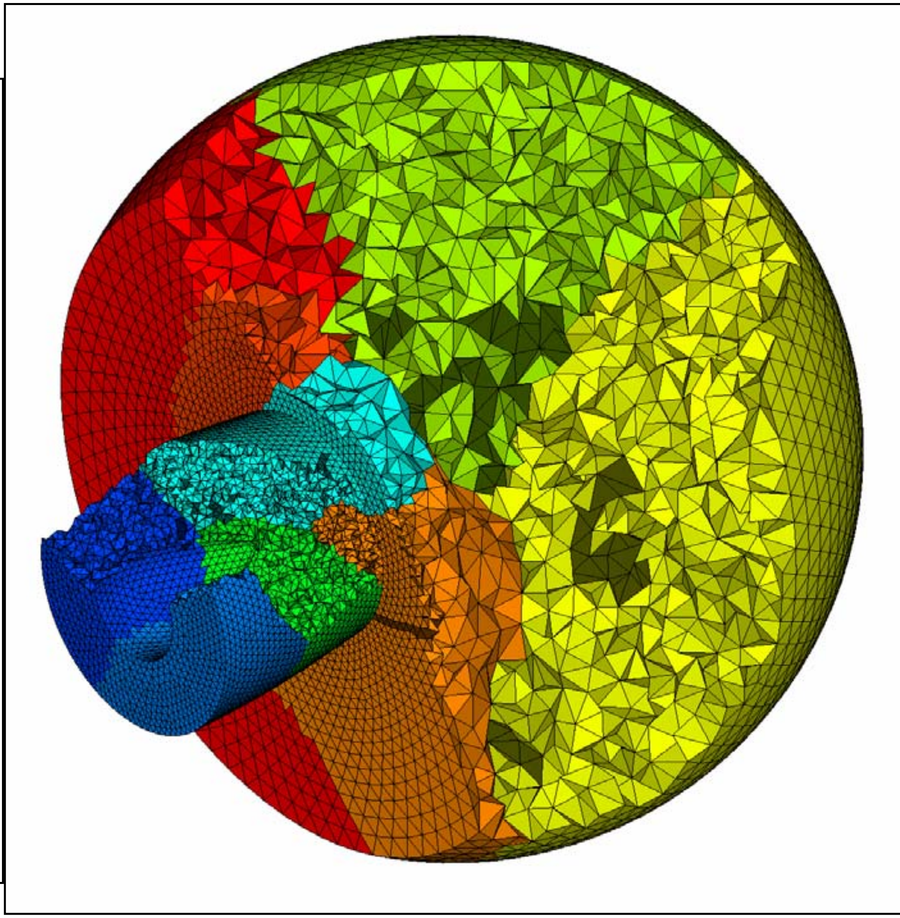
Fan Noise Simulation

- Acoustic wave propagation is modeled with the locally linearized Euler equations (LEE)
- Simulation using 4th order ADER-DG scheme
- 230.000 elements distributed on 8 CPUs
- Background flow is provided by CFD done at TU Berlin
- Fan noise sources are provided by TU Berlin and are modeled as inflow boundary conditions together with a sponge layer technique to allow outgoing waves
- Far field boundaries: characteristic boundary conditions (Riemann solver) together with sponge layer technique
- Farfield is modeled by a sphere intersected with the turbo engine model

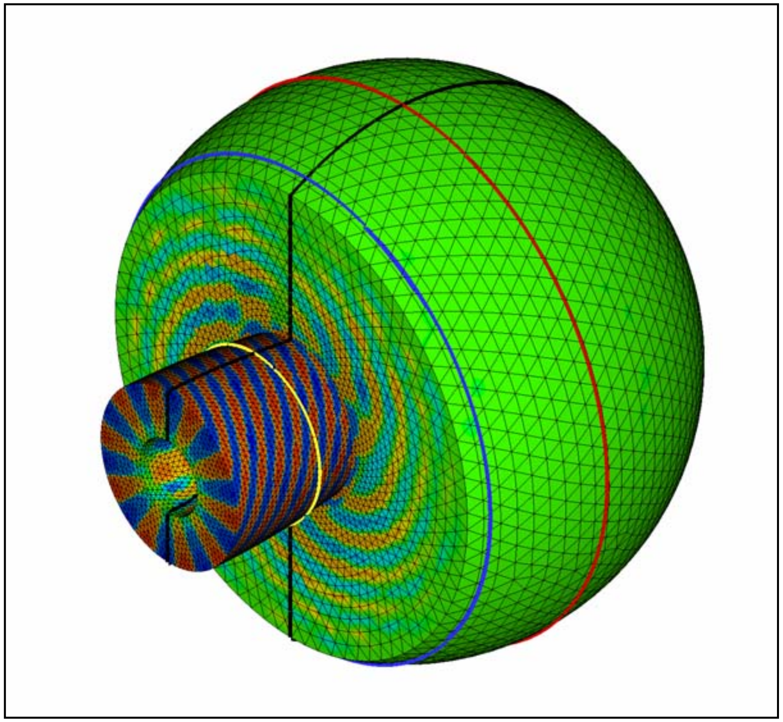
Fan Noise Simulation



Sketch of the turbo engine inlet with fan

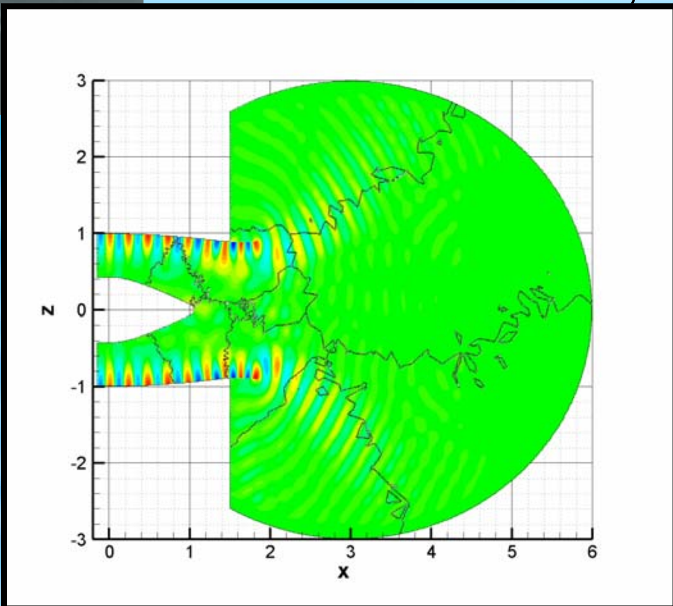


Unstructured CAA mesh distributed on 8 CPUs containing the turbo engine inlet model and the farfield (sphere) acoustic pressure field...

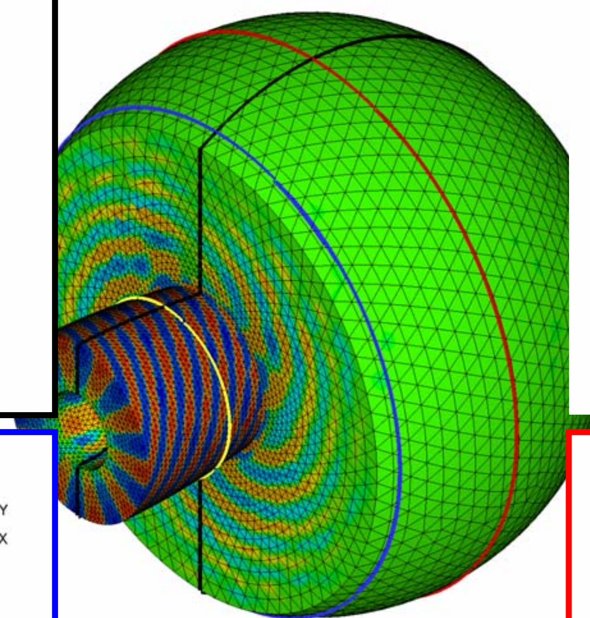


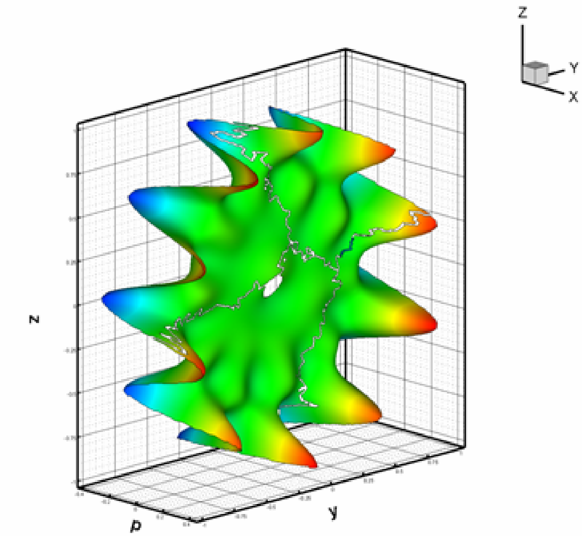
... visualized on the computational mesh

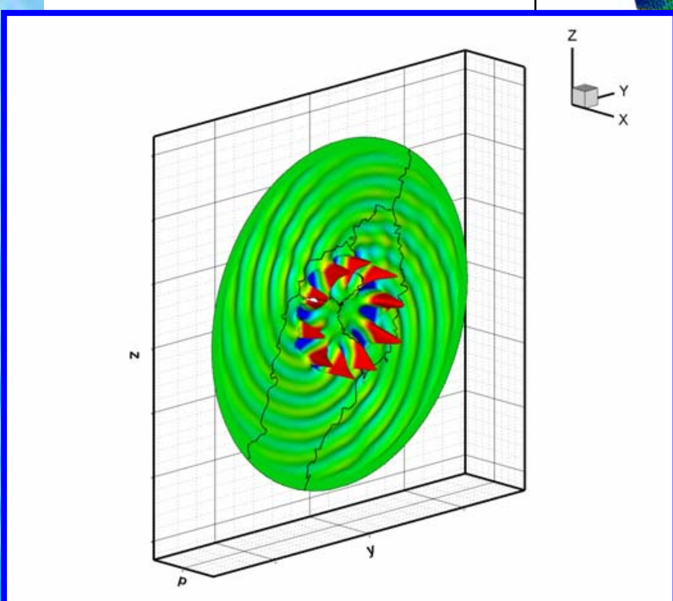
... and on special fine visualization meshes in several cutting planes



acoustic pressure field...

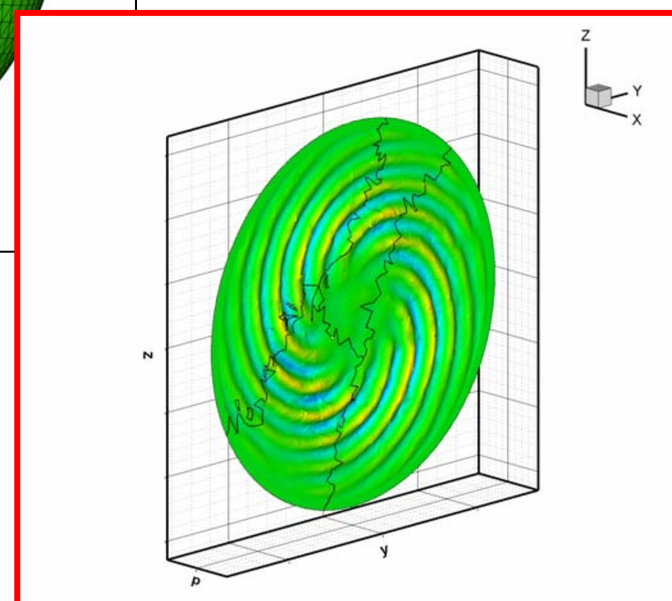






... visualized on the computational mesh

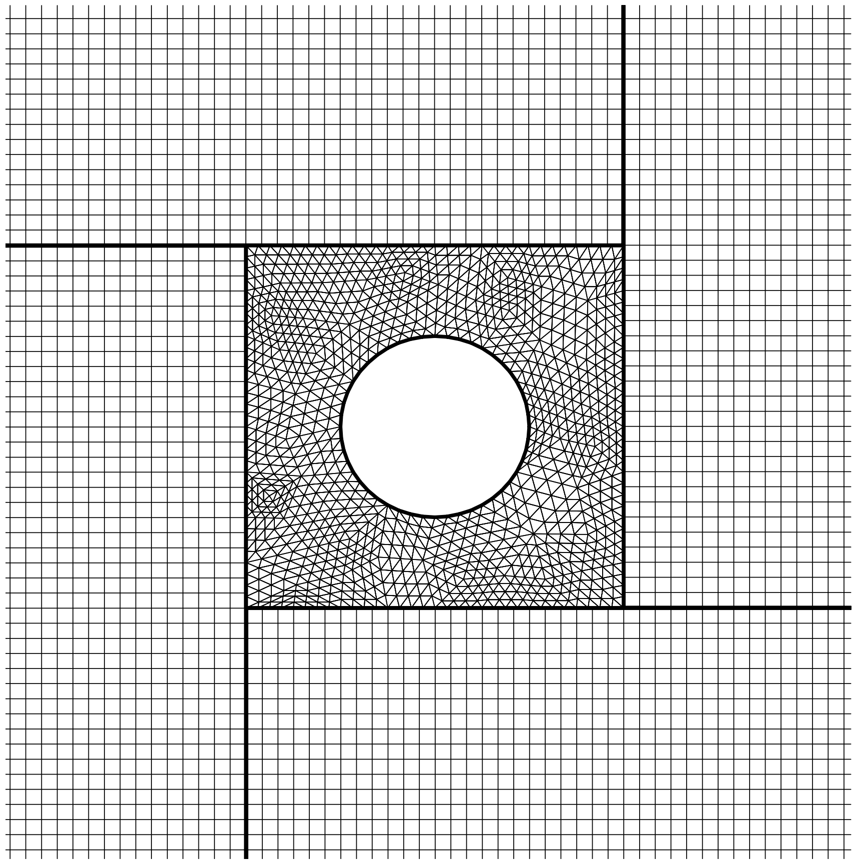
... and on special fine visualization meshes in several cutting planes



Contents

Numerical Examples

Scattering of a Plane Wave at a Cylinder

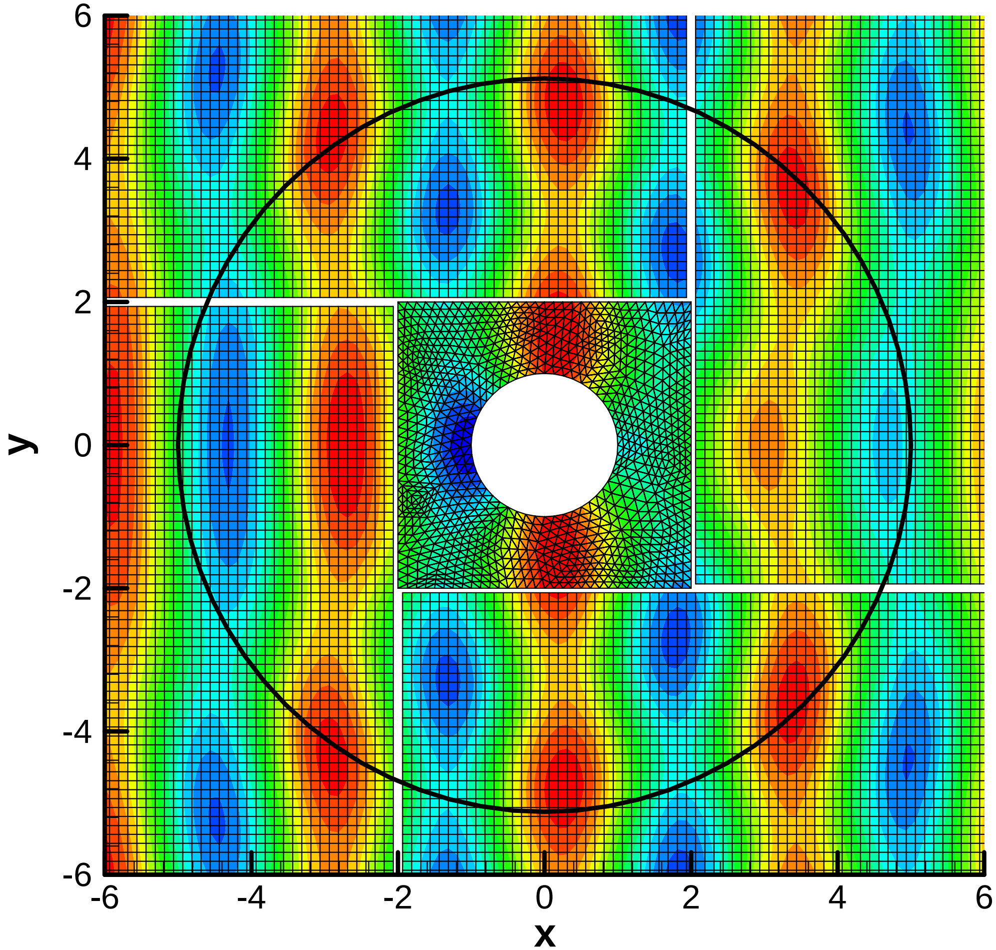


In order to **increase the efficiency** of the calculations:

Coupling of different

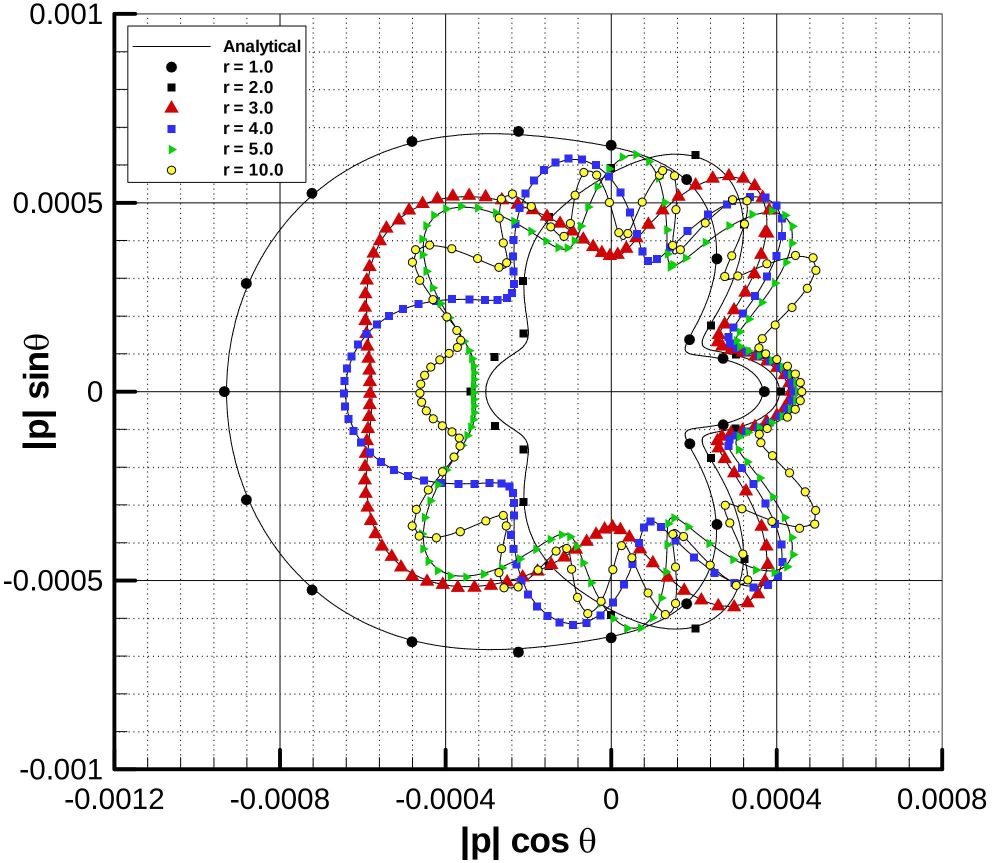
- discretization methods
- mesh sizes
- time steps

At **globally** high order of accuracy !

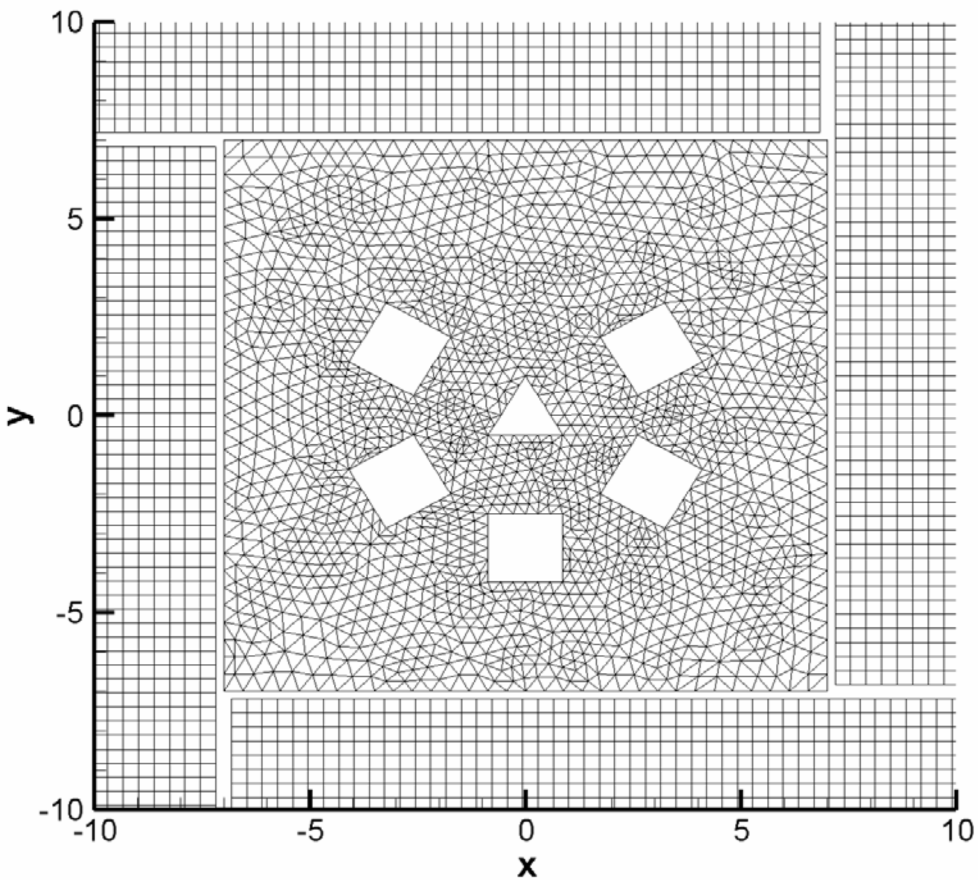


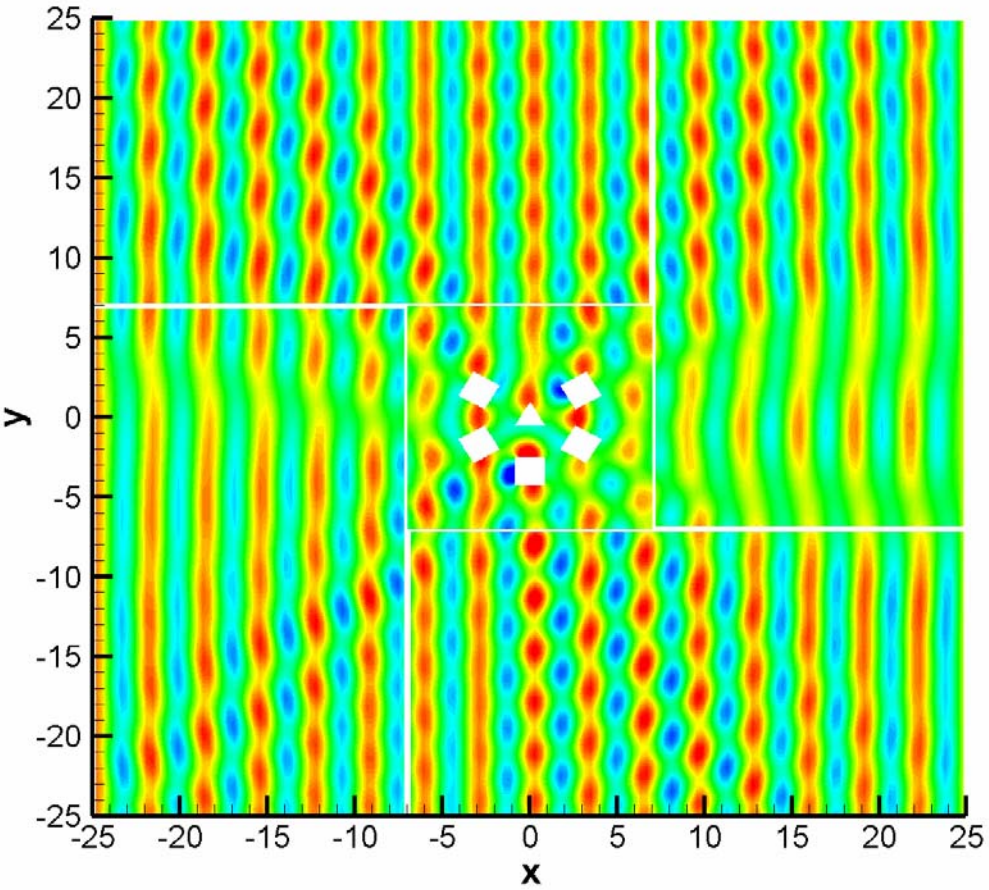
- Scattering of a plane wave (k = 2) on a cylinder (r = 1)
- Linearized Euler Equations,
- 1 unstructured grid: ADER-DG O3
- 4 structured grids: fastADER-FV O10

Directivity Pattern of the Cylinder Scattering Problem
k=2.0, R=1.0



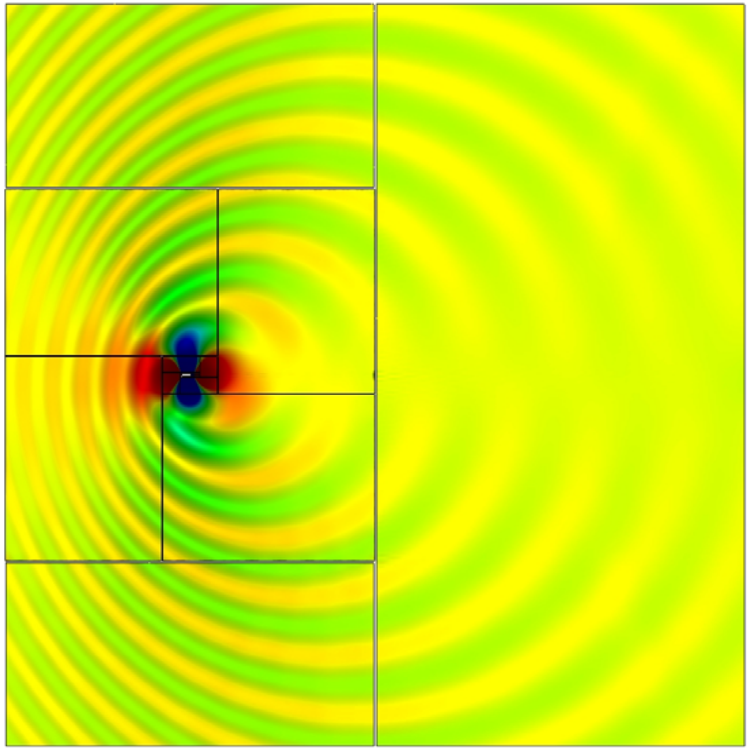
Scattering at Multiple Objects:

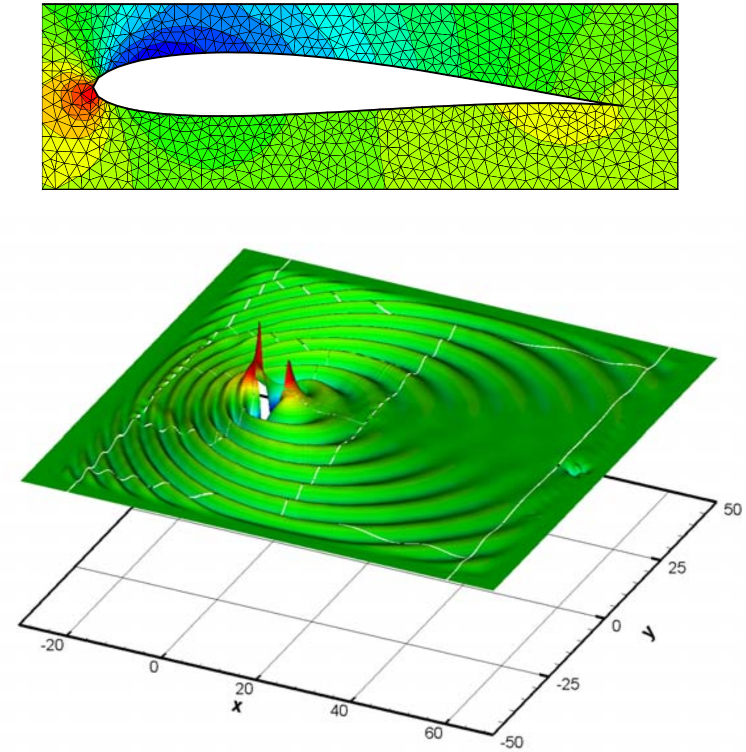



25
20
15
10
5
0
-5
-10
-15
-20
-25
y
-25
-20
-15
-10
-5
0
5
10
15
20
25
x

Sound Generated by an Airfoil in a Vortical Gust

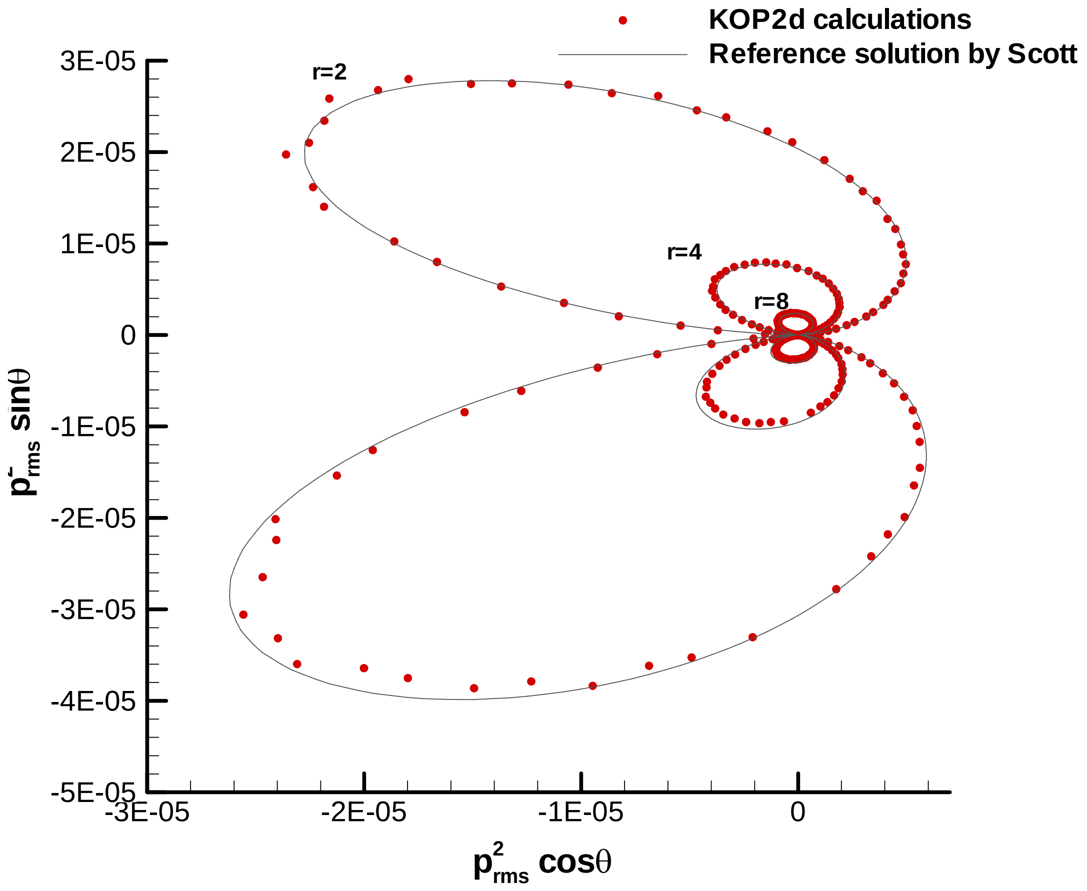
Subsonic Flow: Ma=0.5





Sound field generated by a Joukowski profile, excited by a periodical vortical gust
(AIAA CAA Workshop, Cleveland 2003, benchmark problem 1, category 4, *k*=2)
1 unstructured domain (nonlinear ADER-DG O3), 15 structured domains using 11th order
*fast*ADER for the far field, including 4 sponge layers for the farfield boundary condition

Long wavelength



4. Conclusions

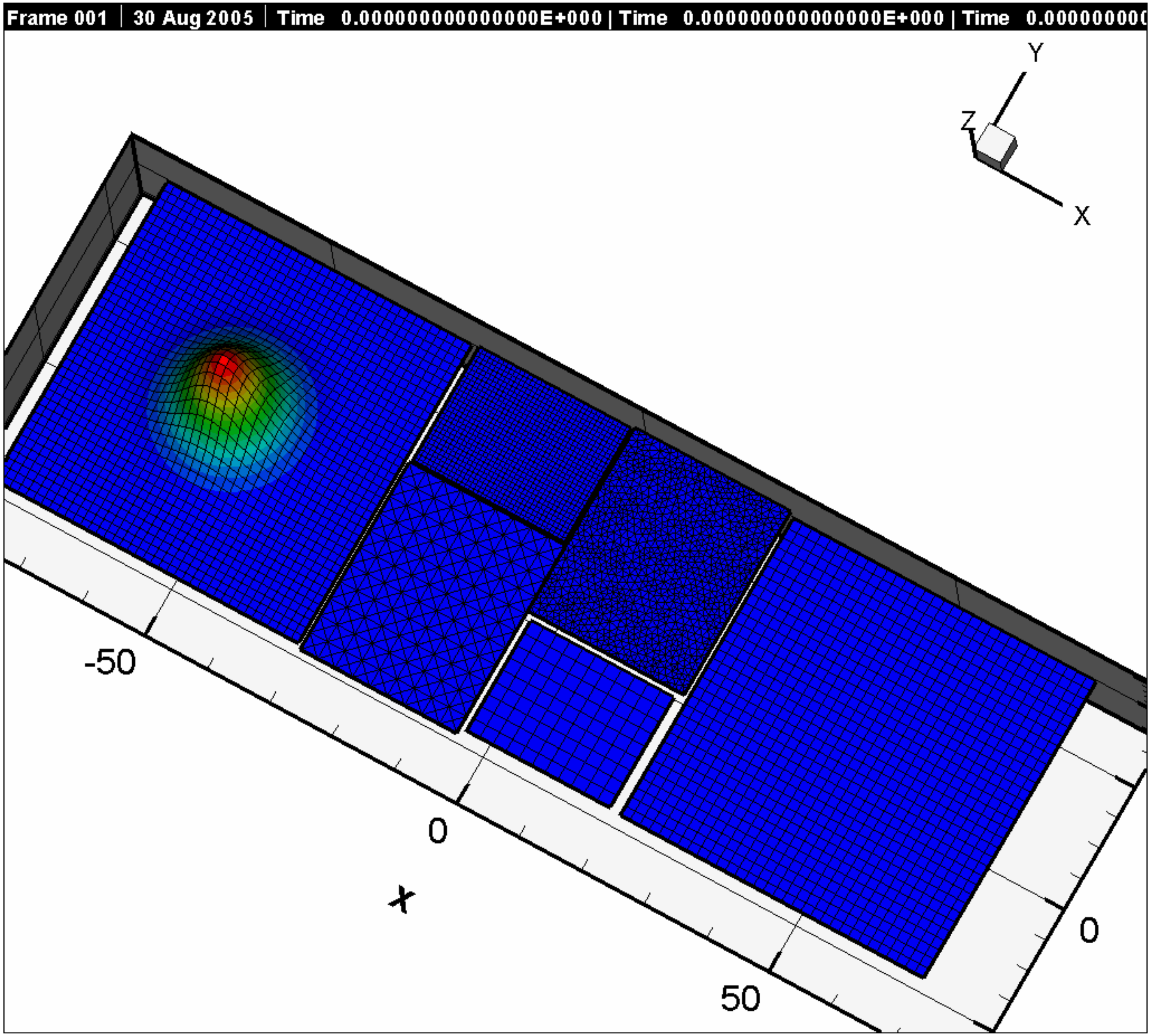
High order schemes: Topic of research

- h-refinement on unstructured grids
- p-refinement: acoustics, boundary layers ...
- Break-through

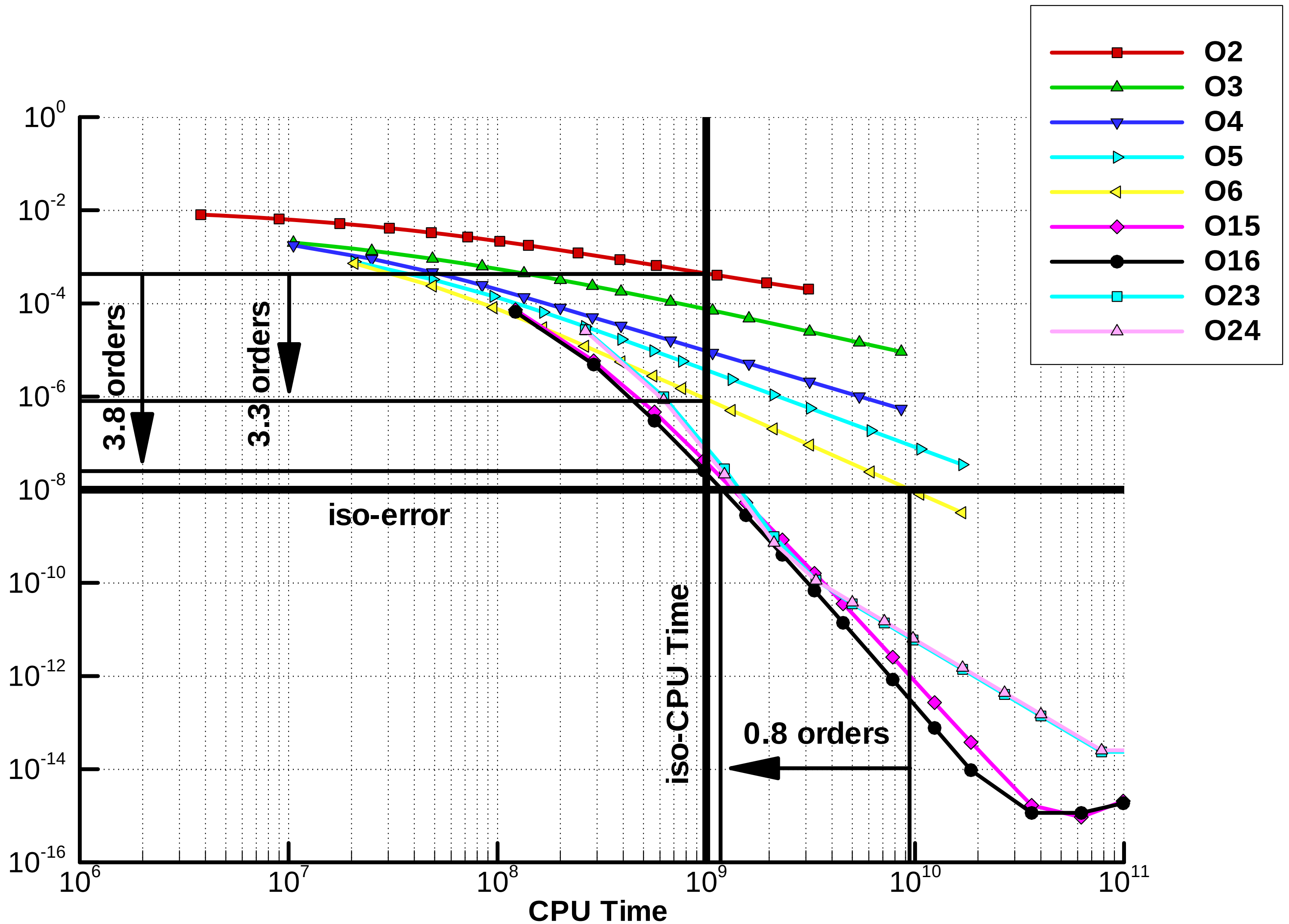
IAG:

- ADER-Finite-Volume on structured grids
- ADER-Discontinuous Galerkin on general grids

Hybrid meshes


Frame 001 | 30 Aug 2005 | Time 0.000000000000000E+000 | Time 0.000000000000000E+000 | Time 0.00000000
Y
Z
X
-50
0
x
50
0

Accuracy


O2
O3
O4
O5
O6
O15
O16
O23
O24
3.8 orders
3.3 orders
iso-error
iso-CPU Time
0.8 orders
CPU Time
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}

1. The Coupling of CFD and Acoustics

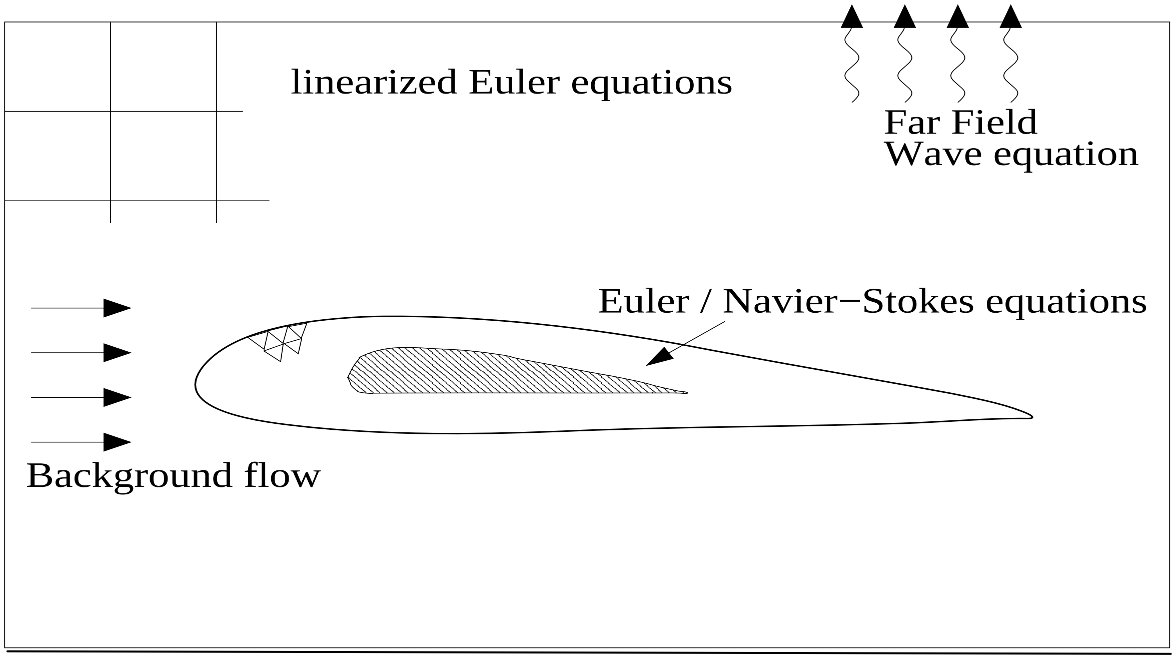
⇨ Hybrid approach

Separate the flow calculation from the acoustics
CFD ⇨ source terms for CAA
volume coupling
CAA: Equations for wave propagation with source terms

⇨ Direct approach

Compressible Navier-Stokes equations
without modeling

Heterogeneous Domain Decomposition



NSE → Euler → linearised Euler → Wave equation

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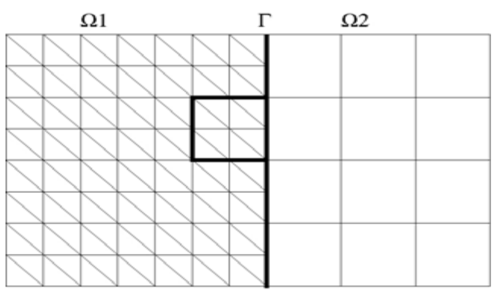
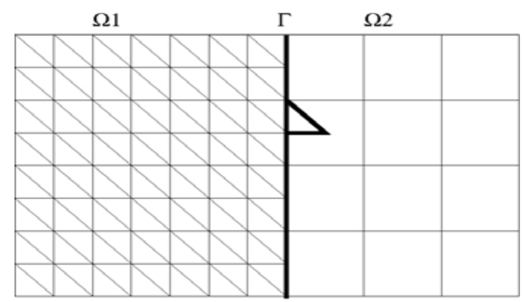
und Geodäsie

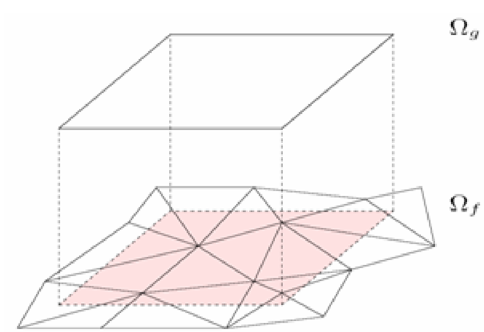
Coupling of Different Space Discretizations

Two Scenarios

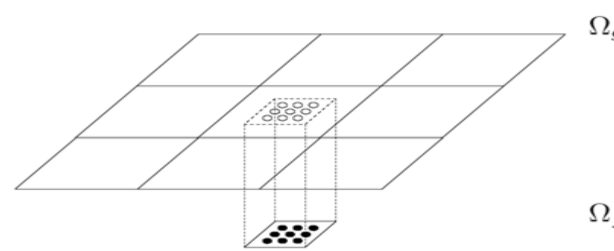




Averaging:
fine ➡ coarse



Interpolation:
coarse ➡ fine

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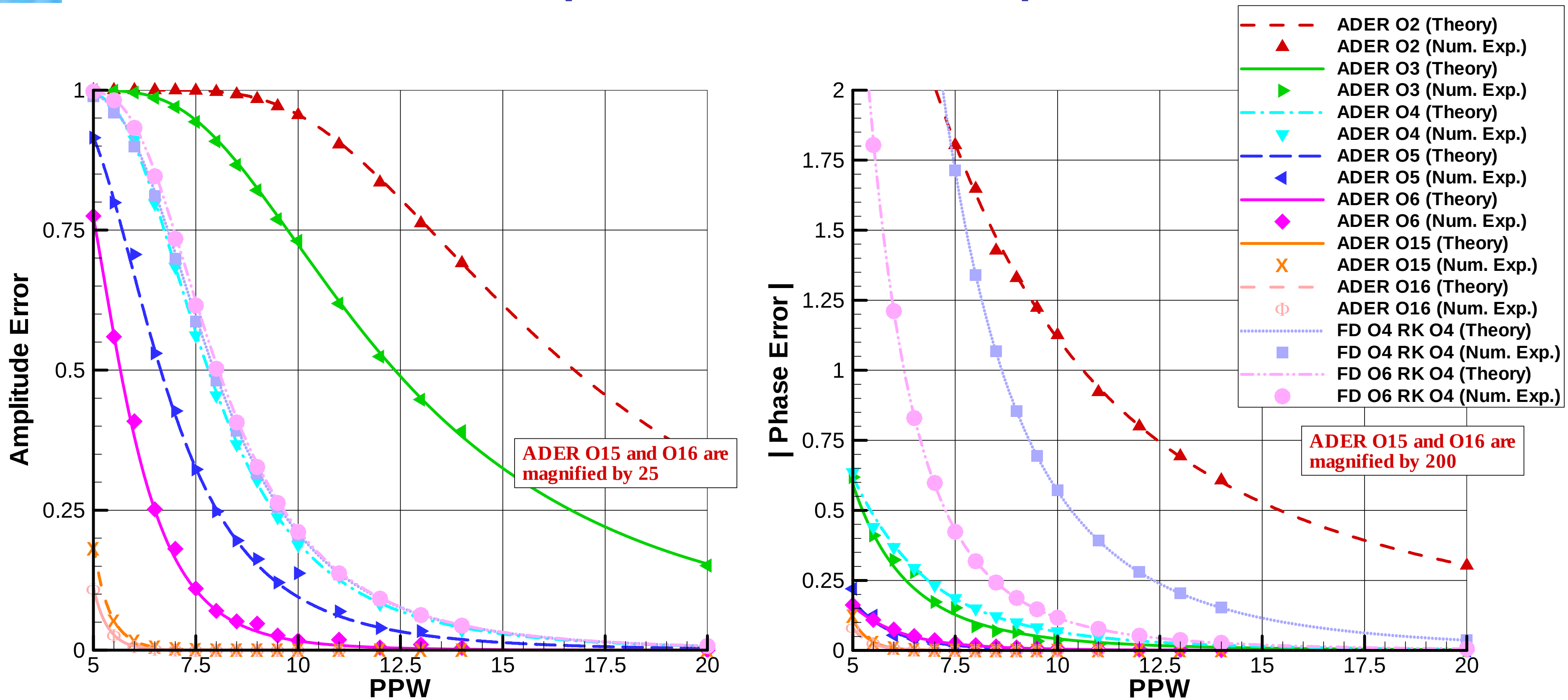
Dissipation and Dispersion



PPW: points per wavelength

Transsonic Flow: Ma=0.9

ENO in non-linear ADER domains for reconstruction