ΑΛΓΟΡΙΘΜΟΣ ΠΑΡΑΓΟΝΤΟΠΟΙΗΣΗΣ SIP/MSIP

$$A_{i,j}\Phi_{i-1,j-1} + B_{i,j}\Phi_{i-1,j} + C_{i,j}\Phi_{i-1,j+1} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1} +$$
$$G_{i,j}\Phi_{i+1,j-1} + H_{i,j}\Phi_{i+1,j} + K_{i,j}\Phi_{i+1,j+1} = q_{i,j}$$

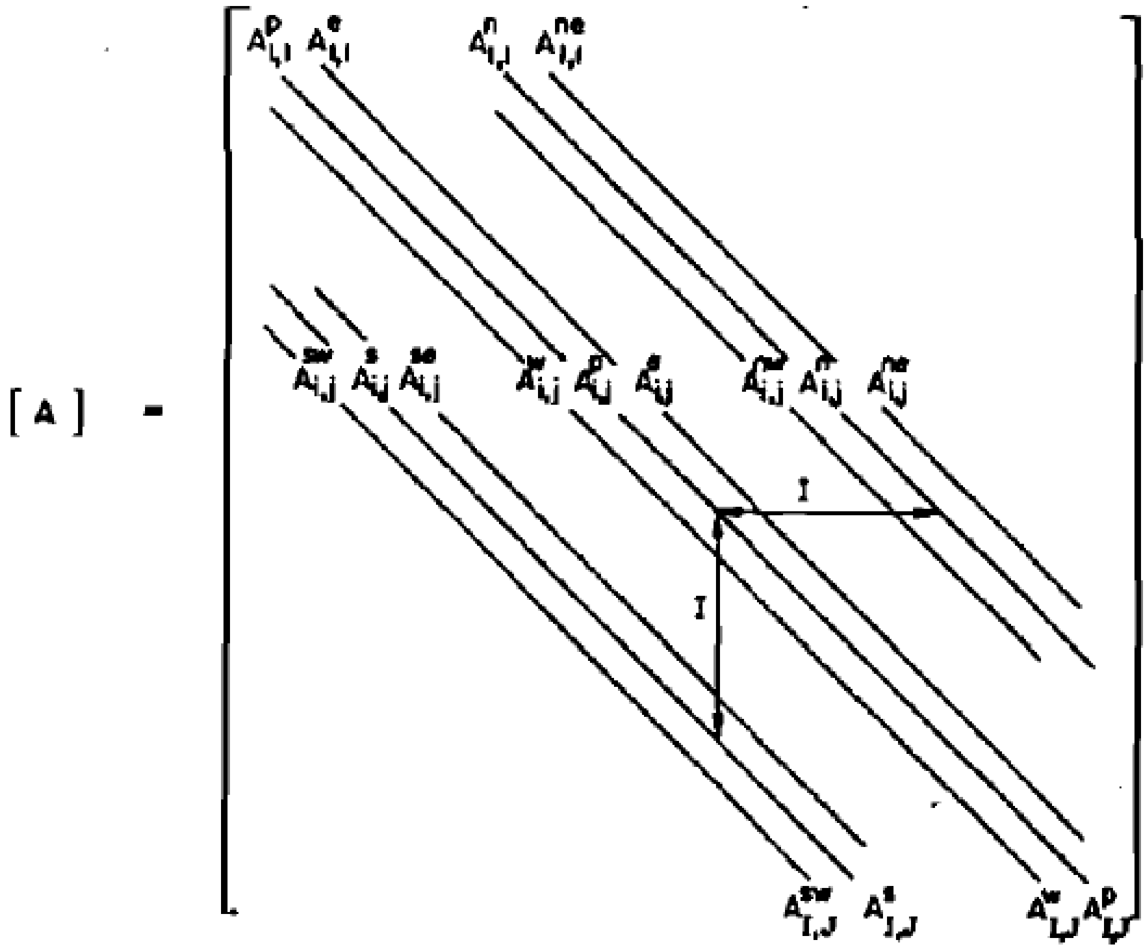
ή AΦ=q.

Stencil κόμβου (i,j):

C	F	K
B	E	H
A	D	G

αντίστοιχο του

i-1,j+1	i,j+1	i+1,j+1
i-1,j	i,j	i+1,j
i-1,j-1	i,j-1	i+1,j-1


[A] =
I
I

E	F			G	H	K																		
D	E	F			G	H	K																	
	D	E	F			G	H	K																
		D	E	F			G	H	K															
C			D	E	F			G	H	K														
B	C			D	E	F			G	H	K													
A	B	C			D	E	F			G	H	K												
	A	B	C			D	E	F			G	H	K											
		A	B	C			D	E	F			G	H	K										
			A	B	C			D	E	F			G	H	K									
				A	B				D	E	F			G	H	K								
					A	B	C			D	E	F			G	H	K							
						A	B	C			D	E	F			G	H	K						
							A	B	C			D	E	F			G	H	K					
								A	B	C			D	E	F			G	H	K				
									A	B	C			D	E	F			G	H	K			
										A	B	C			D	E	F			G	H	K		
											A	B	C			D	E	F			G	H	K	
												A	B	C			D	E	F			G	H	K
													A	B	C			D	E	F			G	H
														A	B	C			D	E	F			G
															A	B	C			D	E	F		
																A	B	C			D	E	F	
																	A	B	C			D	E	F
																		A	B	C			D	E

e																								
d	e																							
	d	e																						
		d	e																					
c			d	e																				
b	c			d	e																			
a	b	c			d	e																		
	a	b	c			d	e																	
		a	b	c			d	e																
			a	b	c			d	e															
				a	b				d	e														
					a	b	c			d	e													
						a	b	c			d	e												
							a	b	c			d	e											
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															a	b	c			d	e			
																a	b	c			d	e		
																	a	b	c			d	e	
																		a	b	c			d	e

1	f			g	h	k																		
	1	f			g	h	k																	
		1	f			g	h	k																
			1	f			g	h	k															
				1	f			g	h	k														
					1	f			g	h	k													
						1	f			g	h	k												
							1	f			g	h	k											
								1	f			g	h	k										
									1	f			g	h	k									
										1	f			g	h	k								
											1	f			g	h	k							
												1	f			g	h	k						
													1	f			g	h	k					
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															1	f			g	h	k			
																1	f			g	h	k		
																	1	f			g	h	k	
																		1	f			g	h	k
																			1	f			g	h
																				1	f			g
																					1	f		
																						1	f	
																							1	f
																								1

Stencil κόμβου (i,j) στο "αρχικό" μητρώο A=LU:

C	F	K
B	E	H
A	D	G

Stencil κόμβου (i,j) στο κάτω τριγωνικό L μητρώο:

c	-	-
b	e	-
a	d	-

Stencil κόμβου (i,j) στο πάνω τριγωνικό U μητρώο:

-	f	k
-	1	h
-	-	g

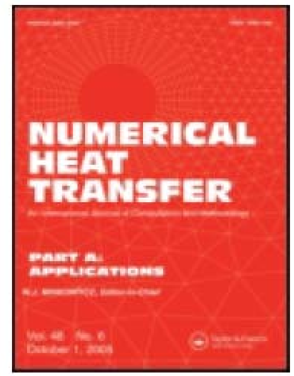
Κανόνας Παραγοντοποίησης SIP ή MSIP

Με την εκτέλεση του πολλαπλασιασμού LU, τα στοιχεία των μη-μηδενικών διαγωνίων του μητρώου A αναπαράγονται ακριβώς.

Από τον πολλαπλασιασμό απομένει "θόρυβος" LU=A+P

Βασική Σχετική Δημοσίευση

A MODIFIED STRONGLY IMPLICIT PROCEDURE FOR THE NUMERICAL SOLUTION OF FIELD PROBLEMS



G. E. Schneider & M. Zedan

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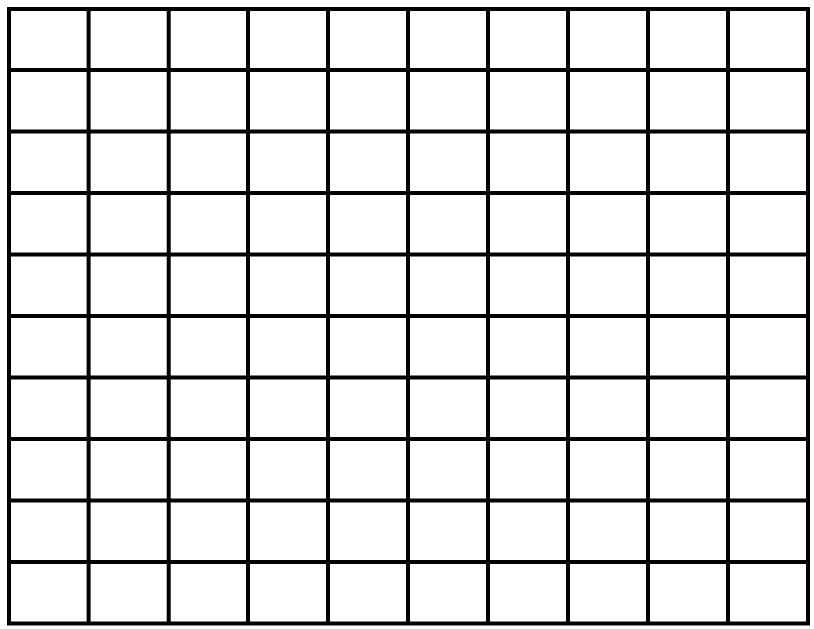
To link to this article: http://dx.doi.org/10.1080/01495728108961775

SIP 5-διαγώνιο stencil

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = q \quad \rightarrow \quad \frac{\Phi_{i+1,j} - 2\Phi_{i,j} + \Phi_{i-1,j}}{\Delta x^2} + \frac{\Phi_{i,j+1} - 2\Phi_{i,j} + \Phi_{i,j-1}}{\Delta y^2} = q_{i,j}$$



$$B_{i,j}\Phi_{i-1,j} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1} + H_{i,j}\Phi_{i+1,j} = q_{i,j}$$

SIP 5-διαγώνιο stencil

Πλέγμα 10x10, Κόμβος (i,j)=(6,5) --> k=55

...	b_{55}	...				...	...		d_{55}	e_{55}
	Σ.45								Σ.54	Σ.55

...	
h_{45}	Γ.45
...	
...	
...	
f_{54}	Γ.54
1	Γ.55

SIP 5-διαγώνιο stencil

Πλέγμα 10x10, Κόμβος (i,j)=(6,5) --> k=55

...	$b_{i,j}$	...				...	...		$d_{i,j}$	$e_{i,j}$
	k-JM								k-1	k

...	
$h_{i-1,j}$	k-JM
...	
...	
...	
$f_{i,j-1}$	k-1
1	k

SIP 5-διαγώνιο stencil

$$\mathbf{B_{i,j}\Phi_{i-1,j} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1} + H_{i,j}\Phi_{i+1,j} = q_{i,j}}$$

	F	
B	**E**	**H**
	D	

Σχέσεις LU Παραγοντοποίησης:

$$\mathbf{b_{i,j} = B_{i,j}}$$

$$\mathbf{d_{i,j} = D_{i,j}}$$

$$\mathbf{e_{i,j} = E_{i,j} - b_{i,j}h_{i-1,j} - d_{i,j}f_{i,j-1}}$$

$$\mathbf{f_{i,j} = F_{i,j} / e_{i,j}}$$

$$\mathbf{h_{i,j} = H_{i,j} / e_{i,j}}$$

SIP 5-διαγώνιο stencil - Εισαγόμενος Θόρυβος

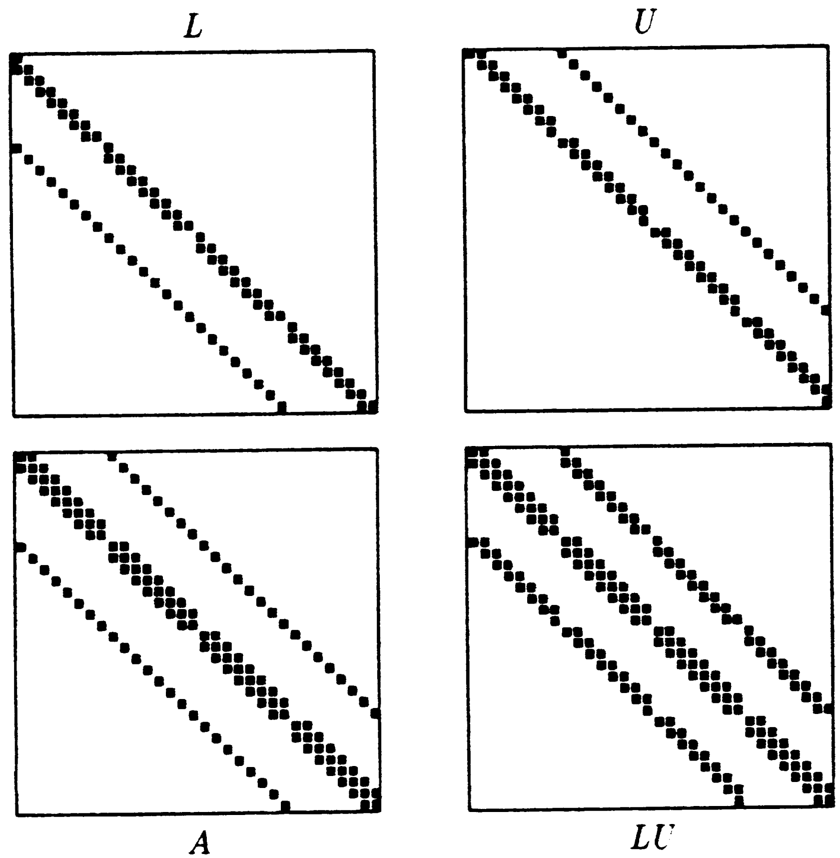
$$\mathbf{B_{i,j}\Phi_{i-1,j} + \pi^1_{i,j}\Phi_{i-1,j+1} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1}}$$
$$\mathbf{+ \pi^2_{i,j}\Phi_{i+1,j-1} + H_{i,j}\Phi_{i+1,j} = q_{i,j}}$$

όπου

$$\pi^1_{i,j} = \mathbf{b_{i,j}f_{i-1,j}}$$
$$\pi^2_{i,j} = \mathbf{d_{i,j}h_{i,j-1}}$$

$\pi^1_{i,j}$	F	
B	E	H
	D	$\pi^2_{i,j}$

SIP   5-διαγώνιο stencil - Εισαγόμενος Θόρυβος



5-διαγώνιο stencil - Η Ιδέα της Modified SIP (MSIP)

Ας προσποιηθούμε ότι παραγοντοποιούμε την εξίσωση:

$$\mathbf{B_{i,j}\Phi_{i-1,j} - \pi^1_{i,j}\Phi_{i-1,j+1} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1}}$$
$$\mathbf{- \pi^2_{i,j}\Phi_{i+1,j-1} + H_{i,j}\Phi_{i+1,j} = q_{i,j}}$$

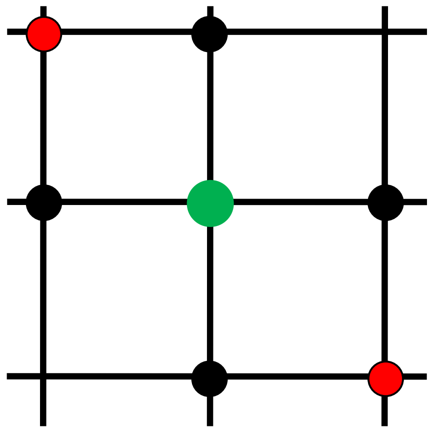
ή, καλύτερα/ασφαλέστερα την:

$$\mathbf{B_{i,j}\Phi_{i-1,j} - \psi\pi^1_{i,j}\Phi_{i-1,j+1} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1}}$$
$$\mathbf{- \psi\pi^2_{i,j}\Phi_{i+1,j-1} + H_{i,j}\Phi_{i+1,j} = q_{i,j}}$$

με $\psi<1$.

SIP 5-διαγώνιο stencil - Η Ιδέα της Modified SIP (MSIP)

Αναπτύγματα Taylor:



$$\Phi_{i-1,j+1} = \Phi_{i-1,j} + \Phi_{i,j+1} - \Phi_{i,j}$$

$$\Phi_{i+1,j-1} = \Phi_{i+1,j} + \Phi_{i,j-1} - \Phi_{i,j}$$

SIP 5-διαγώνιο stencil - Η Ιδέα της Modified SIP (MSIP)

$$\mathbf{B}_{i,j}\Phi_{i-1,j} - \psi\pi^1_{i,j}\Phi_{i-1,j+1} + \mathbf{D}_{i,j}\Phi_{i,j-1} + \mathbf{E}_{i,j}\Phi_{i,j} + \mathbf{F}_{i,j}\Phi_{i,j+1} - \psi\pi^2_{i,j}\Phi_{i+1,j-1} + \mathbf{H}_{i,j}\Phi_{i+1,j} = q_{i,j}$$

$$\mathbf{B}_{i,j}\Phi_{i-1,j} - \psi\pi^1_{i,j}\left(\Phi_{i-1,j} + \Phi_{i,j+1} - \Phi_{i,j}\right) + \mathbf{D}_{i,j}\Phi_{i,j-1} + \mathbf{E}_{i,j}\Phi_{i,j} + \mathbf{F}_{i,j}\Phi_{i,j+1} - \psi\pi^2_{i,j}\left(\Phi_{i+1,j} + \Phi_{i,j-1} - \Phi_{i,j}\right) + \mathbf{H}_{i,j}\Phi_{i+1,j} = q_{i,j}$$

$$\hat{\mathbf{B}}_{i,j}\Phi_{i-1,j} + \hat{\mathbf{D}}_{i,j}\Phi_{i,j-1} + \hat{\mathbf{E}}_{i,j}\Phi_{i,j} + \hat{\mathbf{F}}_{i,j}\Phi_{i,j+1} + \hat{\mathbf{H}}_{i,j}\Phi_{i+1,j} = q_{i,j}$$

όπου:

$$\hat{\mathbf{B}}_{\mathbf{i,j}} = \mathbf{B}_{\mathbf{i,j}} - \psi\pi^{\mathbf{1}}_{\mathbf{i,j}} = \mathbf{B}_{\mathbf{i,j}} - \psi\mathbf{b}_{\mathbf{i,j}}\mathbf{f}_{\mathbf{i-1,j}}$$

$$\hat{\mathbf{D}}_{\mathbf{i,j}} = \mathbf{D}_{\mathbf{i,j}} - \psi\pi^{\mathbf{2}}_{\mathbf{i,j}} = \mathbf{D}_{\mathbf{i,j}} - \psi\mathbf{d}_{\mathbf{i,j}}\mathbf{h}_{\mathbf{i,j-1}}$$

$$\hat{\mathbf{E}}_{\mathbf{i,j}} = \mathbf{E}_{\mathbf{i,j}} + \psi\pi^{\mathbf{1}}_{\mathbf{i,j}} + \psi\pi^{\mathbf{2}}_{\mathbf{i,j}} = \mathbf{E}_{\mathbf{i,j}} + \psi\mathbf{b}_{\mathbf{i,j}}\mathbf{f}_{\mathbf{i-1,j}} + \psi\mathbf{d}_{\mathbf{i,j}}\mathbf{h}_{\mathbf{i,j-1}}$$

$$\hat{\mathbf{F}}_{\mathbf{i,j}} = \mathbf{F}_{\mathbf{i,j}} - \psi\pi^{\mathbf{1}}_{\mathbf{i,j}} = \mathbf{F}_{\mathbf{i,j}} - \psi\mathbf{b}_{\mathbf{i,j}}\mathbf{f}_{\mathbf{i-1,j}}$$

$$\hat{\mathbf{H}}_{\mathbf{i,j}} = \mathbf{H}_{\mathbf{i,j}} - \psi\pi^{\mathbf{2}}_{\mathbf{i,j}} = \mathbf{H}_{\mathbf{i,j}} - \psi\mathbf{d}_{\mathbf{i,j}}\mathbf{h}_{\mathbf{i,j-1}}$$

MSIP 5-διαγώνιο stencil

$$\mathbf{b_{i,j} = B_{i,j} / \left[1 + \psi f_{i-1,j}\right]}$$

$$\mathbf{d_{i,j} = D_{i,j} / \left[1 + \psi h_{i,j-1}\right]}$$

$$\mathbf{e_{i,j} = E_{i,j} - b_{i,j} h_{i-1,j} - d_{i,j} f_{i,j-1} + \psi (b_{i,j} f_{i-1,j} + d_{i,j} h_{i,j-1})}$$

$$\mathbf{f_{i,j} = \left(F_{i,j} - \psi b_{i,j} f_{i-1,j}\right) / e_{i,j}}$$

$$\mathbf{h_{i,j} = \left(H_{i,j} - \psi d_{i,j} h_{i,j-1}\right) / e_{i,j}}$$

Σύγκριση με:

$$\mathbf{b_{i,j} = B_{i,j}}$$

$$\mathbf{d_{i,j} = D_{i,j}}$$

$$\mathbf{e_{i,j} = E_{i,j} - b_{i,j} h_{i-1,j} - d_{i,j} f_{i,j-1}}$$

$$\mathbf{f_{i,j} = F_{i,j} / e_{i,j}}$$

$$\mathbf{h_{i,j} = H_{i,j} / e_{i,j}}$$

SIP 9-διαγώνιο stencil

$$A_{i,j}\Phi_{i-1,j-1} + B_{i,j}\Phi_{i-1,j} + C_{i,j}\Phi_{i-1,j+1} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1} +$$
$$G_{i,j}\Phi_{i+1,j-1} + H_{i,j}\Phi_{i+1,j} + K_{i,j}\Phi_{i+1,j+1} = q_{i,j}$$

Παράδειγμα (σε "απλό" καρτεσιανό πλέγμα):

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial x \partial y} = q \quad \rightarrow$$

$$\frac{\Phi_{i+1,j} - 2\Phi_{i,j} + \Phi_{i-1,j}}{\Delta x^2} + \frac{\Phi_{i,j+1} - 2\Phi_{i,j} + \Phi_{i,j-1}}{\Delta y^2} +$$

$$\frac{\Phi_{i+1,j+1} + \Phi_{i-1,j-1} - \Phi_{i+1,j-1} - \Phi_{i-1,j+1}}{4\Delta x \Delta y} = q_{i,j}$$

SIP 9-διαγώνιο stencil

Πλέγμα 10x10, Κόμβος (i,j)=(6,5) --> k=55

k_{44}	Γ.44
h_{45}	Γ.45
g_{46}	Γ.46
...	
...	
f_{54}	Γ.54
1	Γ.55

a_{55}	b_{55}	c_{55}				...	...		d_{55}	e_{55}
Σ.44	Σ.45	Σ.46							Σ.54	Σ.55

SIP 9-διαγώνιο stencil

Πλέγμα 10x10, Κόμβος (i,j)=(6,5) --> k=55

$a_{i,j}$	$b_{i,j}$	$c_{i,j}$				...	...		$d_{i,j}$	$e_{i,j}$
k-JM-1	k-JM	k-JM+1							k-1	k

$k_{i-1,j-1}$	k-JM-1
$h_{i-1,j}$	k-JM
$g_{i-1,j+1}$	k-JM+1
...	
...	
$f_{i,j-1}$	k-1
1	k

SIP 9-διαγώνιο stencil

$$a_{i,j} = A_{i,j}$$

$$b_{i,j} = B_{i,j} - a_{i,j} f_{i-1,j-1}$$

$$c_{i,j} = C_{i,j} - b_{i,j} f_{i-1,j}$$

$$d_{i,j} = D_{i,j} - a_{i,j} h_{i-1,j-1} - b_{i,j} g_{i-1,j}$$

$$e_{i,j} = E_{i,j} - a_{i,j} k_{i-1,j-1} - b_{i,j} h_{i-1,j} - c_{i,j} g_{i-1,j+1} - d_{i,j} f_{i,j-1}$$

$$f_{i,j} = \left(F_{i,j} - b_{i,j} k_{i-1,j} - c_{i,j} h_{i-1,j+1}\right) / e_{i,j}$$

$$g_{i,j} = \left(G_{i,j} - d_{i,j} h_{i,j-1}\right) / e_{i,j}$$

$$h_{i,j} = \left(H_{i,j} - d_{i,j} k_{i,j-1}\right) / e_{i,j}$$

$$k_{i,j} = K_{i,j} / e_{i,j}$$

SIP 9-διαγώνιο stencil - Εισαγόμενος Θόρυβος

$$\begin{aligned} &A_{i,j}\Phi_{i-1,j-1} + B_{i,j}\Phi_{i-1,j} + C_{i,j}\Phi_{i-1,j+1} + \pi^1_{i,j}\Phi_{i-1,j+2} \\ &+ \pi^2_{i,j}\Phi_{i,j-2} + D_{i,j}\Phi_{i,j-1} + E_{i,j}\Phi_{i,j} + F_{i,j}\Phi_{i,j+1} + \pi^3_{i,j}\Phi_{i,j+2} \\ &+ \pi^4_{i,j}\Phi_{i+1,j-2} + G_{i,j}\Phi_{i+1,j-1} + H_{i,j}\Phi_{i+1,j} + K_{i,j}\Phi_{i+1,j+1} = q_{i,j} \end{aligned}$$

όπου

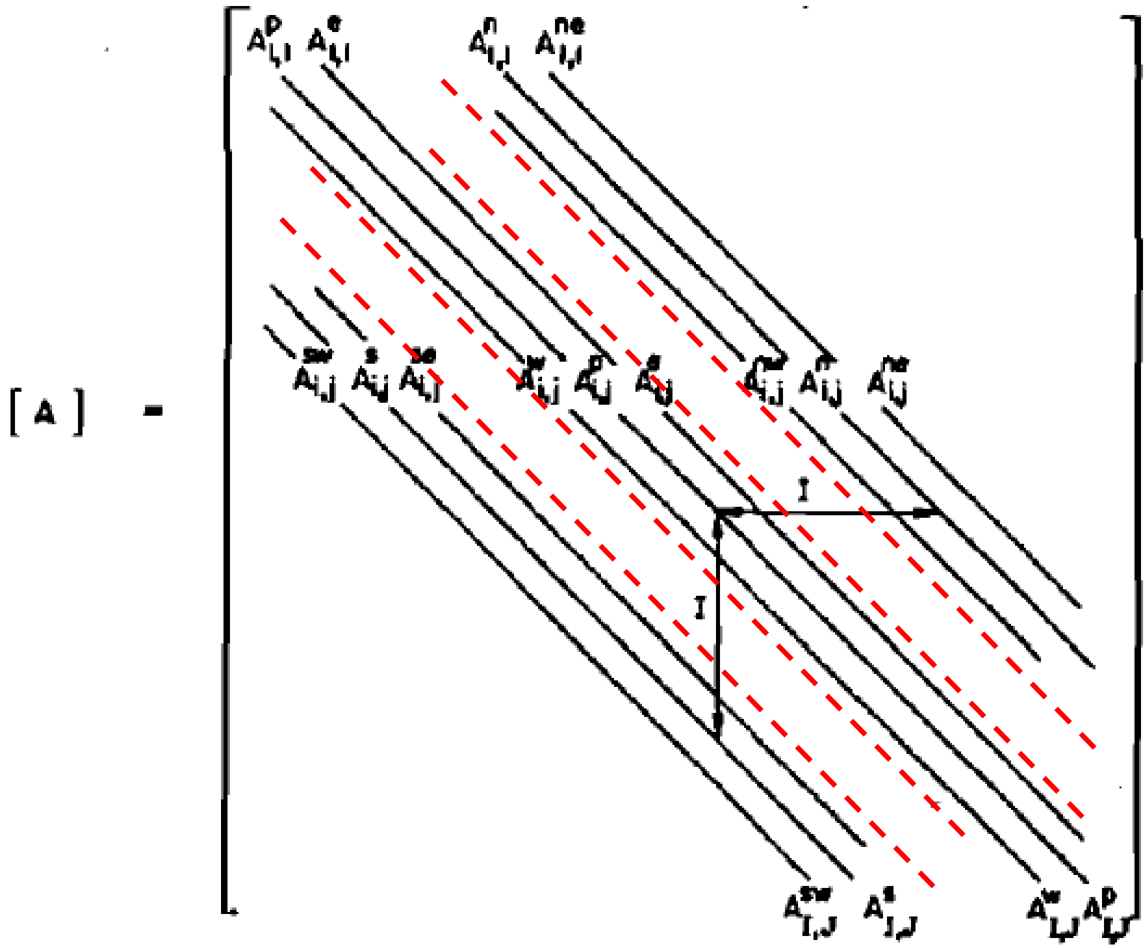
$$\pi^1_{i,j} = c_{i,j} f_{i-1,j+1}$$

$$\pi^2_{i,j} = a_{i,j} g_{i-1,j-1}$$

$$\pi^3_{i,j} = c_{i,j} k_{i-1,j+1}$$

$$\pi^4_{i,j} = d_{i,j} g_{i,j-1}$$

$\pi^1_{i,j}$	$\pi^3_{i,j}$	
C	F	K
B	E	H
A	D	G
	$\pi^2_{i,j}$	$\pi^4_{i,j}$


[A] =
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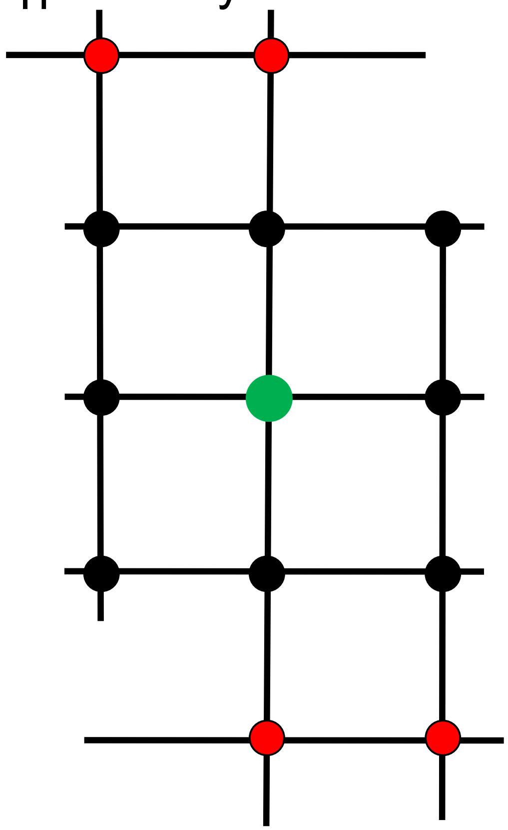
9-διαγώνιο stencil - Η Ιδέα της Modified SIP (MSIP)

Αναπτύγματα Taylor:



$$\Phi_{i-1,j+2} = \Phi_{i-1,j} + 2\Phi_{i,j+1} - 2\Phi_{i,j}$$

$$\Phi_{i,j-2} = 2\Phi_{i,j-1} - \Phi_{i,j}$$

$$\Phi_{i,j+2} = 2\Phi_{i,j+1} - \Phi_{i,j}$$

$$\Phi_{i+1,j-2} = \Phi_{i+1,j} + 2\Phi_{i,j-1} - 2\Phi_{i,j}$$

MSIP 9-διαγώνιο stencil

$$a_{i,j} = A_{i,j}$$

$$b_{i,j} = \left[B_{i,j} - \psi f_{i-,j+1} C_{i,j} - a_{i,j} f_{i-1,j-1}\right] / \left[1 - \psi f_{i-1,j} f_{i-1,j+1}\right]$$

$$c_{i,j} = C_{i,j} - b_{i,j} f_{i-1,j}$$

$$d_{i,j} = \left[D_{i,j} - 2\psi a_{i,j} g_{i-1,j-1} - a_{i,j} h_{i-1,j-1} - b_{i,j} g_{i-1,j}\right] / \left[1 + 2\psi g_{i,j-1}\right]$$

$$e_{i,j} = E_{i,j} - a_{i,j} k_{i-1,j-1} - b_{i,j} h_{i-1,j} - c_{i,j} g_{i-1,j+1} - d_{i,j} f_{i,j-1} +$$
$$+ 2\psi(c_{i,j} f_{i-1,j+1} + d_{i,j} g_{i,j-1}) + \psi(a_{i,j} g_{i-1,j-1} + c_{i,j} k_{i-1,j+1})$$

$$f_{i,j} = \left(F_{i,j} - 2\psi c_{i,j}[f_{i-1,j+1} + k_{i-1,j+1}] - b_{i,j} k_{i-1,j} - c_{i,j} h_{i-1,j+1}\right) / e_{i,j}$$

$$g_{i,j} = \left(G_{i,j} - d_{i,j} h_{i,j-1}\right) / e_{i,j}$$

$$h_{i,j} = \left(H_{i,j} - \psi d_{i,j} g_{i,j-1} - d_{i,j} k_{i,j-1}\right) / e_{i,j}$$

$$k_{i,j} = K_{i,j} / e_{i,j}$$

Άσκηση

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = q$$

$$\left.\frac{\partial^2 \Phi}{\partial x^2}\right|_i = \frac{-\Phi_{i+2} + 16\Phi_{i+1} - 30\Phi_{i,j} + 16\Phi_{i-1} - \Phi_{i-2}}{12\Delta x^2} + O(\Delta x^4)$$