INVERSE DESIGN OF AERODYNAMIC SHAPES USING ANT COLONY OPTIMIZATION

T. Zervogiannis, V. Assouti,
K. Gagas, A. Kaounis and K.C. Giannakoglou
National Technical University of Athens,
P.O. Box 64069, Athens 157 10, GREECE.
e-mail :kgianna@central.ntua.gr

1 SUMMARY

In the past, Ant Colony Optimization methods have been used to solve combinatorial optimization problems, like the well known Traveling Salesman Problem. This paper introduces a novel extension of the conventional method which is capable of solving optimization problems with continuous search spaces for the free variables. For this purpose, various notions which are implicit to the Ant Colony Optimization techniques are modified in conformity with the particularities of search problems in continuous spaces. The proposed method will be used for the design of aerodynamic shapes, isolated airfoils or compressor cascade airfoils.

2 INTRODUCTION

The Ant Colony Optimization (ACO) method is a recently proposed metaheuristic. In the past, it has been used to solve stochastic combinatorial optimization problems, [1], [2], such as the Traveling Salesman Problem (TSP) or routing problems. In all these problems, integer numbers should be arranged in the proper order which ensures the minimization of a functional. In the (ACO) method, ants stand for agents with search capabilities similar to those of real ants. They act synergetically, i.e. in populations, seeking for the optimum route. A new population of ants comes after the previous one and inherits coded information about the quality of previously evaluated routes. Information is communicated in the form of pheromone trails laid down by the previous populations. Pheromone trails affect ants' decisions about where to go next, with a probability which is proportional to the amount of pheromone.

This paper aims to extend the ACO methods' capabilities to new scientific areas, such as the inverse or optimum design of aerodynamic shapes. In the past, other optimization methods, such as Genetic Algorithms (GAs), have been widely used for the design of ducts of airfoils, [3]. The aim is to design shapes along which the pressure (or velocity) distribution at given flow conditions matches a target one. The shape parameterization, which points out the design variables, is an evident prerequisite.

Computational Fluid Dynamics (*CFD*) tools are employed for the evaluation of candidate solutions. In contrast to combinatorial problems, the inverse design of a shape (*IDS*) is a problem with continuous search spaces for the design variables and a likely complex solution landscape.

The use of ACO in this kind of problems is, in fact, novel. To the authors knowledge, this could be the first use of ACO in a problem with real (continuous) free parameters. The concept is simple; provided that we do possess an effective ACO method (and the relevant software) for solving the *TSP* problem, modifications should be employed so as to transform the the *IDS* problem to an equivalent *TSP*. The route length, which is the cost function in the *TSP*, is taken to be the deviation of the pressure or velocity distribution along the shape contour from the target one.

For the readers which are not familiar with the ACO method, this will be presented below at length. Then, its novel implementation to the *IDS* problem will be described, followed by indicative results.

3 THE *ACO* METHOD

Ants, though almost blind animals, are capable of finding the shortest path from their nest to areas rich in food. Many individuals, perhaps almost the entire ant colony, should cooperate to achieve this goal. In the search of the shortest route to the food, ants use to exploit pheromone trails which are laid on the ground by each one of them as they move. Pheromone is a substance easily traced by subsequent ants, which are likely to follow the trail rather than move randomly. When an ant follows an existing pheromone trail, the latter will be further reinforced by its own pheromone. Thus, a frequently used trail becomes more attractive and is likely to be followed by a continuously increasing number of ants. The ants' behaviour in search of food is the concept of the so-called ACO method. This is a population-based algorithm, where pheromone trails define the feedback of information between ants. Since moving ants act independently, ACO is readily amenable to parallelization.

The amount of pheromone a moving agent lays on the ground can be computed in various ways. We will present one of them on the basis of the *TSP* problem. In the *TSP*, given n towns (with known coordinates) the salesman should visit all of them once and then return to the starting town, with the minimum route length.

A possible way for solving the *TSP* using ACO is through as many ants as the number of towns (n agents). Each ant starts its route from a different town. The next town to be visited is chosen with a probability p_{ij} that depends on the distance d_{ij} between the current i and the next j town to be visited (already visited towns are excluded) and the amount of pheromone τ_{ij} laid on the connecting edge. This probability is expressed as follows:

$$p_{ij} = \frac{\tau_{ij}^{\alpha} d_{ij}^{-\beta}}{\sum_j \tau_{ij}^{\alpha} d_{ij}^{-\beta}} \tag{1}$$

The inverse of the distance between two towns (d_{ij}^{-1}) is usually referred to as visibility. Upon completion of n (closed) tours by the current population agents, the pheromone trails τ_{ij} are updated as $\tau_{ij}^{new} = \rho\tau_{ij}^{old} + \Delta\tau_{ij}$, where $(1-\rho)$ is the evaporation coefficient.

The pheromone quantity $\Delta\tau_{ij}$, contributed by the k-th ant which rounded off its tour with length D_k is $\Delta\tau_{ij} \propto D_k^{-1}$ if the k-th ant used edge (i, j), otherwise $\Delta\tau_{ij} = 0$.

4 INVERSE DESIGN OF AIRFOILS

The inverse design of airfoils can be envisaged as a minimization problem which employes: *(a)* one or more targets; here the target is to achieve the desired pressure coefficient distribution over the airfoil walls, at given flow conditions, *(b)* the parameterization of the airfoil contour; Bezier-Bernstein polynomials are used where the ordinates of the Bezier points are the design variables of degrees of freedom (*dofs*), *(c)* the evaluation tool; incompresible fluids and irrotational flows are assumed, modelled through a simple and non-costly tool, i.e.. the panel method, [4]; however, the method may readily incorporate more accurate tools (Navier-Stokes or Euler equations solvers), in a straightforward manner, *(d)* the optimization tool, which in our case is a novel ACO method, as it will be described in the next paragraph.

5 THE EXTENDED *ACO* ALGORITHM

Let the airfoil contour be described using two Bezier curves, one for the pressure side (from the trailing to the leading edge, with n_1 control points) and the other for the suction side (in the opposite direction, with n_2 control points). Leading and trailing edge points are kept fixed. Fixed is also the abscissa of any other control point giving rise to $n = n_1 + n_2 - 4$ *dofs*. In the example of fig. 1 (left), $n_1 = 5$, $n_2 = 5$ and $n = 6$; vertical lines indicate the search space for the ordinates of the n Bezier control points. Let also assume that the combined effects of the Bezier polynomial and flow solver on the n *dofs*, $\overrightarrow{Y} = (y_1, ..., y_n)$, yields a curve (i.e. the pressure distribution) that should minimize its deviation from the target curve. Both curves are shown in fig. 1 (right), for the airfoil pressure side. In this example, the payoff value associated with $\overrightarrow{Y}$ is the area enclosed by the two curves (plus the corresponding area for the suction side).

Under the previous assumptions, we should first cast the *IDS* problem illustrated in fig. 1 as an equivalent *TSP* (to be referred to as $eTSP$), which will then be solved through the modified ACO algorithm. In the $eTSP$ the salesman should visit n territories, instead of n towns. Ants should mimic this itinerary. The territories associated with each *dof* are the vertical lines in fig. 1. The sequence of the territories to be visited is known and the tour is closed. The optimization method should locate the sequence of visiting points over each territory so that the total path be of minimum length. For the path followed by an ant, the term "length" is used metaphorically and stands for the cost associated with the corresponding airfoil. To compute this cost, the evaluation software described in the previous paragraph should be used.

In the ACO algorithm, eq. 1, the probability with which the next destination is chosen by an ant depends on two parameters, namely the distance d and the pheromone trail τ. The former represents a sort of *local* data (correlating the actual position of an agent and the towns that are likely to be visited next) whereas the latter stands for *global* information (i.e. information related to the evaluation of $\overrightarrow{Y}$ as a whole). Local data can be defined in many ways. Here, we will assume that the relative position of the k^{th} and $(k+1)^{th}$ Bezier points is associated with the hatched area shown in fig. 1. This

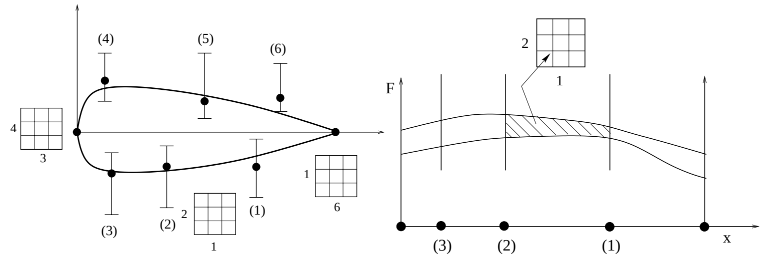

Figure 1: Design of airfoils using Bezier points. Global and local costs and the grids for storing probabilities.

can readily be generalized for all *dofs*. So, the global cost associated with an agent tour and the corresponding airfoil shape splits to partial or "local" costs using separators that can either be defined by the user or set automatically (vertical separators in fig. 1, right). A noticeable feature of the proposed method is that information is stored in a discrete way, though it is used as a continuous field. One grid for storing local (i.e. distances) and another one for storing global (feromone trails) data are needed for each pair of consecutive territories (i.e. *dofs*), fig. 1.

A cycle corresponds to n tours with n agents, each starting from a different territory. The closed tour by any agent corresponds to n values $\overrightarrow{Y} = (y_1, ..., y_n)$ and yields an airfoil shape. Through its evaluation, global and local (or partial) costs are computed. These are pieces of information that should be stored over the aforementioned grids. On each grid, for instance the k^{th} one, the y_k and y_{k+1} values are used to locate a point on this grid. The $\tau_{k,k+1}$ and $d_{k,k+1}$ values are computed as the inverse of the global and the corresponding local cost, respectively. Then, all grid nodes are given a feromone and distance value by employing a $2D$ exponential decay based on distances measured over the grid. At the end of the cycle, pheromone is allowed to evaporate, using a law similar to the one employed in the standard ACO. Probabilities are stored over the same nodes by post-processing the stored pheromone and distance values.

The exploitation of the probabilities stored over the grid nodes during the search is simple. The k^{th} agent starts from a point (not necessarily coinciding with a grid node) on the k^{th} grid, determined in a probabilistic way on the basis of the nodal probability values. Then, each agent moves in the clockwise direction. Its next station on the next territory can be found by entering the grid with the abscissa (known from the previous station) and by computing the ordinate in a probabilistic way.

6 RESULTS

First, an assessment of the present method in the conventional *TSP* problem will be given. In the Eilon's 50-town problem, [5], the best computed route is shown in fig. 2. The minimum computed route length is equal to 428.1. This is similar to results exposed in other works, [5], by means of ACO or GAs.

The redesign of the symmetric isolated $NACA12$ airfoil, at zero and 10^o incidence as well as the redesign of the $NACA65$ cascade airfoil (stagger angle= 30^o, solidity=1.25, inlet flow angle= 48^o) have been analyzed using the proposed method. Results are presented in figs. 3, 4 and 5. Using the known airfoil shapes and the panel method

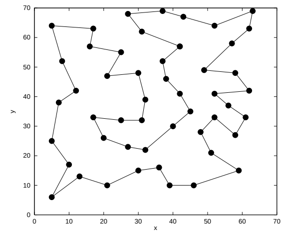

Figure 2: Optimum solution to the Eilon's 50-town *TSP*.

solver, we have first computed the pressure distributions over their contours, at the aforesaid flow conditions. These have been used as target distributions, so the sought for airfoil shapes were known beforehand. In all cases, $\alpha = \beta = 0.8$ and $\rho = 0.5$. In the $NACA12$ case, $n_1 = n_2 = 8$ whereas in the $NACA65$ one $n_1 = 9$ and $n_2 = 8$.

As it may be seen from the comparison of the best predicted and target distributions (or even by comparing starting and predicted airfoils), our method gives very satisfactory results. The cost of the three redesigns is illustrated in fig 6. In this figure, the abscissa stands for calls to the evaluation *CFD* tool which determines the computing cost of the optimization method. To avoid all misunderstanding, we should point out that the use of the panel method leads to almost negligible computing cost per evaluation, but the last figure is a very good indication of the computing cost in case of a more *CPU*-demanding *CFD* method (eg. Navier-Stokes solver).

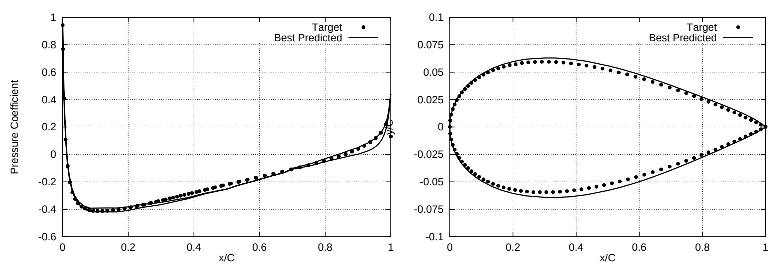

Figure 3: Redesign of the $NACA12$ isolated profile at zero incidence: target and best computed pressure coefficient distribution along with the corresponding airfoil shapes.

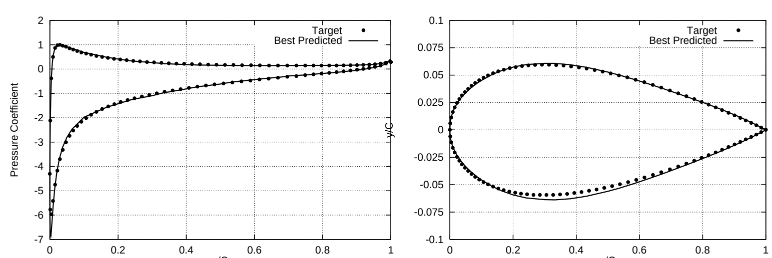

Figure 4: Redesign of the $NACA12$ isolated profile at 10^o incidence: target and best computed pressure coefficient distribution along with the corresponding airfoil shapes.

7 CONCLUSIONS

A new method has been proposed for optimization problems involving degrees of freedom that may vary over continuous search spaces. Such a typical problem is the inverse

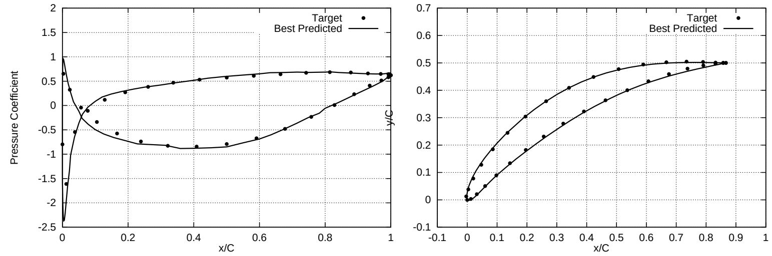

Figure 5: Redesign of the $NACA66$ compressor cascade: target and best computed pressure coefficient distribution along with the corresponding airfoil shapes.

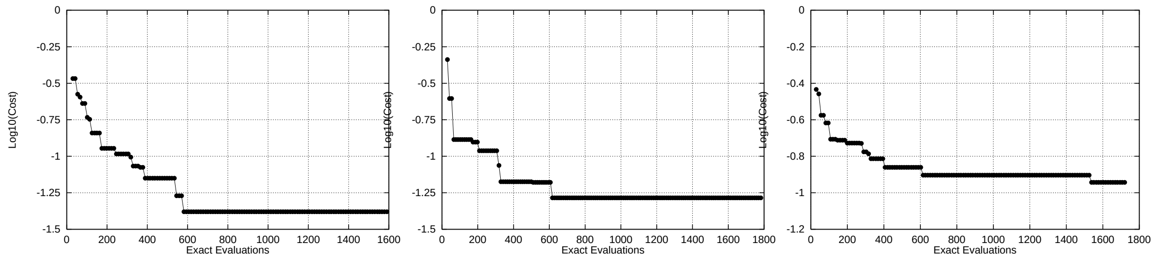

Figure 6: Convergence histories.

design of aerodynamic shapes. The new method is based on the Ant Colony Optimization algorithm, used so far only for combinatorial optimization problems. The new method required the definition of quantities that mimic any quantity that appears in the traditional ACO method. In this respect, global and local costs have been defined and used after being stored on properly defined grids. The method proved to be an effective optimization tool for the inverse design of isolated airfoils or cascades.

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