EVOLUTIONARY METHODS FOR DESIGN, OPTIMIZATION AND CONTROL
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CONTINUOUS ADJOINT METHOD WITH LOW–MACH NUMBER PRECONDITIONING

Varvara G. Asouti⋆, Dimitrios I. Papadimitriou, Kyriakos C. Giannakoglou
Parallel CFD & Optimization Unit, Lab. of Thermal Turbomachines,
National Technical University of Athens (NTUA),
P.O. Box 64069, Athens 15710, Greece
⋆ *email: vasouti@mail.ntua.gr*

Abstract. This paper presents a continuous adjoint formulation for inverse design problems in aerodynamics, based on the low–Mach number preconditioned flow equations. An existing adjoint method, which yields gradient expressions free of field integrals is extended to the incompressible flow regime using the compressible flow equations. The proposed method is applied to the inverse designs of isolated and cascade airfoils in inviscid and viscous low speed flows where the use of preconditioning reduces considerably the CPU cost.

Key words: Aerodynamic shape optimization, adjoint methods, low–Mach number preconditioning

1 INTRODUCTION

During the last decades, computational methods for aerodynamic shape optimization problems have reached a certain level of maturity. Among them, the gradient–based optimization techniques use the derivatives of the objective function with respect to the design variables to iteratively update design variables by pointing to the local direction of improvement. In the so–called adjoint method, basically inspired by control theory[1], the objective function derivatives are computed by solving the adjoint equations with almost equal cost to that of solving the flow equations, irrespective of the number of design variables.

In "conventional" adjoint formulations, the gradient expression contains field integrals[1] which are costly to compute and may introduce inaccuracies. For inviscid flows, Jameson[2] proposed a development that skips the computation of field coordinate sensitivities. A more general approach (with extension to viscous flows) has been proposed[3] by these authors and is the basis of the present method.

It is known that the main reason for the performance degradation of time–marching compressible flow (and adjoint) solvers is the large disparity between acoustic waves and fluid speeds. By preconditioning[4,5] the flow equations one may overcome this problem. In view of the above, the objective of this paper is to extend an existing adjoint method[3] to incompressible flows using an all–speed flow solver[6]. The use of preconditioned flow and adjoint equations reduces considerably the optimization cost at very low speeds. Aerodynamic shapes are parameterized using Bézier curves with N design variables $(b_1, ..., b_N)$. The adjoint method computes $\frac{\delta F}{\delta b_i}$, where F is the objective function and the steepest descent method $b_i^{new} = b_i^{old} - \eta \frac{\delta F}{\delta b_i}$ is used to update the designed shapes.

2 NAVIER–STOKES & LOW–MACH PRECONDITIONING

The preconditioned flow equations of a compressible fluid are written as follows

$$\Gamma^{-1} \frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{f}_i^{inv}}{\partial x_i} - \frac{\partial \vec{f}_i^{vis}}{\partial x_i} = 0 \tag{1}$$

where $\vec{U}$ is the vector of conservative variables and $\vec{f}_i^{inv}$, $\vec{f}_i^{vis}$ are the inviscid and viscous fluxes, respectively. The preconditioning matrix Γ depends on $a = min(1, M^2)$, where M is the local Mach number. Γ is defined by[5]

$$\Gamma = \left(\frac{\partial \vec{U}}{\partial \vec{V}} \right) \begin{bmatrix} 1 & 0 & 0 & -\frac{1-a}{c^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a \end{bmatrix} \left(\frac{\partial \vec{U}}{\partial \vec{V}} \right)^{-1} \tag{2}$$

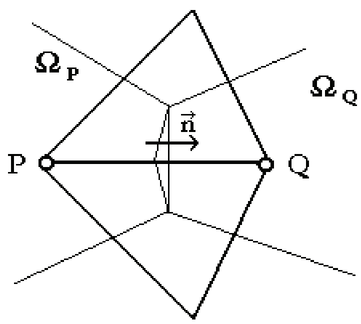


Figure 1: The 1D Riemann problem for the convective fluxes between nodes P and Q. Finite volumes Ω_P and Ω_Q are shown.

where $\left(\frac{\partial \vec{U}}{\partial \vec{V}} \right)$ is the tranformation matrix from non–conservative variables $\overrightarrow{V}$ to $\overrightarrow{U}$. Eqs. 1 are discretized on unstructured grids with triangular elements, using the vertex–centered finite volume technique. This results to the balance of numerical fluxes $\overrightarrow{\Phi}$ crossing the boundary $\partial \Omega_P$, of any volume Ω_P

$$\frac{\Omega_P}{\Delta t_P} \Delta \vec{U}_P + \Gamma_P \sum_{Q \in nei(P)} \overrightarrow{\Phi}_{PQ} = 0 \tag{3}$$

where $nei(P)$ denotes the set of nodes linked to P by a grid edge, $\overrightarrow{\Phi}_{PQ}$ is the flux crossing the interface of finite volumes Ω_P and Ω_Q and $\Delta \vec{U}_P = \vec{U}_P^{\kappa+1} - \vec{U}_P^{\kappa}$ (κ stands for the iteration counter). A $1D$ Roe approximate Riemann solver[7](fig. 1) is used to compute the inviscid flux $\overrightarrow{\Phi}_{PQ}^{inv}$ according to

$$\overrightarrow{\Phi}_{PQ}^{inv} = \frac{1}{2} \left[A_P \vec{U}_P + A_Q \vec{U}_Q \right] - \frac{1}{2} \tilde{\Gamma}_{PQ}^{-1} \, |\tilde{A}_\Gamma|_{PQ} \, \Delta \vec{U}_{PQ} \tag{4}$$

where $A_i = \frac{\partial \vec{f}_i}{\partial \vec{U}}$, $A_P = A_{i_P} n_{PQ,i}$, $A_Q = A_{i_Q} n_{PQ,i}$, $|\tilde{A}_\Gamma|_{PQ} = |\tilde{\Gamma}_{PQ} \tilde{A}_{PQ}|$ is the Roe–averaged preconditioned Jacobian at midnode and $\Delta \vec{U}_{PQ} = \vec{U}_Q - \vec{U}_P$. The assumption $|\tilde{\Gamma}_{PQ}^{-1} \, \tilde{\Gamma}_{PQ} \, \tilde{A}_{PQ}| \simeq \tilde{\Gamma}_{PQ}^{-1} \, |\tilde{\Gamma}_{PQ} \tilde{A}_{PQ}|$ is made. Viscous fluxes are computed by assuming that $\overrightarrow{V}$ undergoes a linear distribution within each triangular element.

3 ADJOINT FORMULATION USING PRECONDITIONING

The continuous adjoint approach for the inverse design of aerodynamic shapes at very low flow speeds, based on the preconditioned Navier–Stokes equations as state equations, is developed. Our aim is to design isolated and cascade airfoils that produce a given pressure distribution $p_{tar}(S)$ over their solid walls S_w, at specified flow conditions. The corresponding objective or cost function is

$$F = \frac{1}{2}\int_{S_w}(p - p_{tar})^2 dS \tag{5}$$

and its variation due to variations in the design variables becomes

$$\delta F = \tfrac{1}{2}\int_{S_w}(p - p_{tar})^2\delta(dS) + \int_{S_w}(p - p_{tar})\delta p dS \tag{6}$$

where $\delta(dS) = \phi_1(\delta\vec{b})$ depends on the parameterization. Based on the preconditioned flow equations, the variation in the augmented objective function is written as

$$\delta F_{aug} = \delta F + \int_\Omega \vec{\Psi}^T \delta\left[\Gamma\left(\tfrac{\partial \vec{f_i}}{\partial x_i}\right)\right] d\Omega = \delta F + \int_\Omega \vec{\Psi}^T \Gamma\ \delta\left(\tfrac{\partial \vec{f_i}}{\partial x_i}\right) d\Omega \tag{7}$$

where $\vec{f_i} = \vec{f}_i^{inv} - \vec{f}_i^{vis}$. Note that the last expression in eq. 7 has been derived by considering $\frac{\partial \vec{f_i}}{\partial x_i} = 0$. Since the variation in the gradient of any quantity Φ can be expressed in terms of the gradient of $\delta\Phi$ and variations in nodal coordinates[5]

$$\delta\left(\tfrac{\partial \vec{\Phi}}{\partial x_i}\right) = \tfrac{\partial(\delta\vec{\Phi})}{\partial x_i} - \tfrac{\partial\vec{\Phi}}{\partial x_k}\tfrac{\partial(\delta x_k)}{\partial x_i} \tag{8}$$

we obtain

$$\int_\Omega \vec{\Psi}_\Gamma^T\ \delta\left(\tfrac{\partial \vec{f_i}}{\partial x_i}\right) d\Omega = \int_\Omega \vec{\Psi}_\Gamma^T\ \tfrac{\partial(\delta\vec{f_i})}{\partial x_i} d\Omega - \int_\Omega \vec{\Psi}_\Gamma^T\ \tfrac{\partial \vec{f_i}}{\partial x_k}\tfrac{\partial(\delta x_k)}{\partial x_i} d\Omega \tag{9}$$

where $\vec{\Psi}_\Gamma = \Gamma^T\vec{\Psi}$ should be referred to as the vector of the preconditioned adjoint variables. It is a matter of integration by parts to get

$$\int_\Omega \vec{\Psi}_\Gamma^T\ \tfrac{\partial(\delta\vec{f_i})}{\partial x_i} d\Omega = -\int_\Omega \delta\vec{f}_i^T \tfrac{\partial \vec{\Psi}_\Gamma^T}{\partial x_i} d\Omega + \int_{S_{i,o,w}} \vec{\Psi}_\Gamma^T\ \delta\vec{f_i} n_i dS \tag{10}$$

and

$$-\int_\Omega \vec{\Psi}_\Gamma^T\ \tfrac{\partial \vec{f_i}}{\partial x_k}\tfrac{\partial(\delta x_k)}{\partial x_i} d\Omega = \int_\Omega \tfrac{\partial}{\partial x_i}\left(\vec{\Psi}_\Gamma^T\ \tfrac{\partial \vec{f_i}}{\partial x_k}\right)\delta x_k d\Omega - \int_{S_w} \vec{\Psi}_\Gamma^T\ \tfrac{\partial \vec{f_i}}{\partial x_k}\delta x_k n_i dS \tag{11}$$

where S_i and S_o are the inlet and outlet boundaries and $\delta(x_k) = \phi_2(\delta\vec{b})$. Further development of eqs. 10 and 11 for the inviscid and viscous terms is omitted, in the interest of space. After several mathematical rearrangements and the satisfaction of the adjoint equations, we obtain

$$\begin{aligned}\delta F =& \tfrac{1}{2}\int_{S_w}(p - p_{tar})^2\delta(dS) + \int_{S_w}\left[\vec{\Psi}_{\Gamma i+1}p - \vec{\Psi}_\Gamma^T\vec{f}_i^{inv}\right]\delta(n_i dS)\\ -& \int_{S_w}\tfrac{\partial \vec{U}^T}{\partial x_k}A_n{}^T\vec{\Psi}_\Gamma\delta x_k dS + \int_{S_w}\vec{\Psi}_\Gamma\tfrac{\partial \vec{f}_i^{vis}}{\partial x_i}\delta x_k\ n_i\ dS + \int_{S_w}\vec{\Psi}_{\Gamma 4}q_i\delta(n_i dS)\\ +& \int_{S_w}\tfrac{\vec{\Psi}_{\Gamma i+1}}{n_i}\tau_{ij}\delta(n_i n_j)dS - \int_{S_w}\tfrac{\partial u_i}{\partial x_k}\tau_{ij}^{(\Psi_\Gamma)}\delta x_k n_j dS - \int_{S_w}\tfrac{\partial T}{\partial x_k}\ k\ \tfrac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_i}\delta x_k\ n_i\ dS\end{aligned} \tag{12}$$

where $\delta(n_i dS) = \phi_3(\delta\vec{b})$ and $\delta(n_i n_j) = \phi_4(\delta\vec{b})$. Using eq. 12 with appropriate functions ϕ_1 to ϕ_4 (depending on the parameterization) the gradient $\frac{\delta F}{\delta b_i} = \frac{\delta F_{aug}}{\delta b_i}$ can be computed and used in the steepest descent method. $\tau_{ij}^{(\Psi_\Gamma)}$ are the so–called "adjoint stresses", given by

$$\tau_{ij}^{(\Psi_\Gamma)} = \mu_{eff}\left(\frac{\partial \vec{\Psi}_{\Gamma j+1}}{\partial x_i} + u_j \frac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_i} + \frac{\partial \vec{\Psi}_{\Gamma i+1}}{\partial x_j} + u_i \frac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_j}\right) - \frac{2}{3}\mu_{eff}\delta_{ij}\left(\frac{\partial \vec{\Psi}_{\Gamma k+1}}{\partial x_k} + u_k \frac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_k}\right)$$

with $\mu_{eff} = \mu + \mu_t$ being the effective viscosity.

The adjoint variables are computed by solving the field preconditioned adjoint equations. After taking Γ^T out of the spatial derivative, they are written as

$$\frac{\partial \vec{\Psi}}{\partial t} - A_{\Gamma_i}^T \frac{\partial \vec{\Psi}}{\partial x_i} - \left(\frac{\partial \vec{U}}{\partial \vec{V}}\right)^{-T} \vec{K} = 0 \tag{13}$$

where $\vec{K}$ is defined as $(i=1,2)$

$$K_1 = -\frac{T}{\varrho}\frac{\partial}{\partial x_j}\left(k\frac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_j}\right), \; K_{i+1} = \frac{\partial \tau_{ij}^{(\Psi_\Gamma)}}{\partial x_j} - \tau_{ij}\frac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_j}, \; K_4 = \frac{T}{p}\frac{\partial}{\partial x_j}\left(k\frac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_j}\right)$$

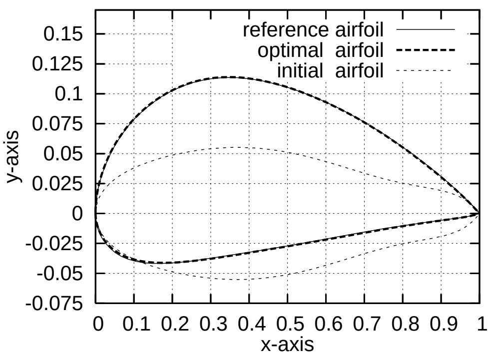
The inlet–outlet boundary conditions are defined by eliminating the integrals of $\delta\vec{U}$ over the inlet and outlet $(\delta\vec{U}^T(A_n^T\vec{\Psi}_\Gamma) = 0)$ whereas, along the solid walls, the condition for the adjoint variables that correspond to the velocity components is $\vec{\Psi}_{\Gamma i+1} = -(p - p_{tar})n_i, i = 1, 2$. For constant wall temperature or adiabatic flows, $\vec{\Psi}_{\Gamma 4} = 0$ or $\frac{\partial \vec{\Psi}_{\Gamma 4}}{\partial x_i} n_i = 0$, respectively.

The integration of the preconditioned adjoint equations, eq. 13, over any finite volume (as defined in section 2) gives the adjoint flux as follows

$$\vec{\Phi}_{PQ}^{\Psi} = \frac{1}{2}\left(-A_{\Gamma_P}^T \vec{\Psi}_P - A_{\Gamma_Q}^T \vec{\Psi}_Q\right) - \frac{1}{2}|\tilde{A}_\Gamma^T|_{PQ}\left(\vec{\Psi}_Q - \vec{\Psi}_P\right) \tag{14}$$

4 RESULTS–DISCUSSION

The proposed method is demonstrated using two airfoil design problems. For the parameterization of the airfoils, two Bézier curves are used separately for the pressure and suction sides.



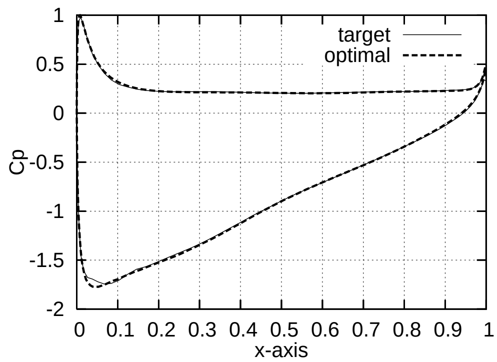


Figure 2: Case I: Initial, reference and optimal airfoil contour (not in scale, left) and the corresponding pressure distributions (right).

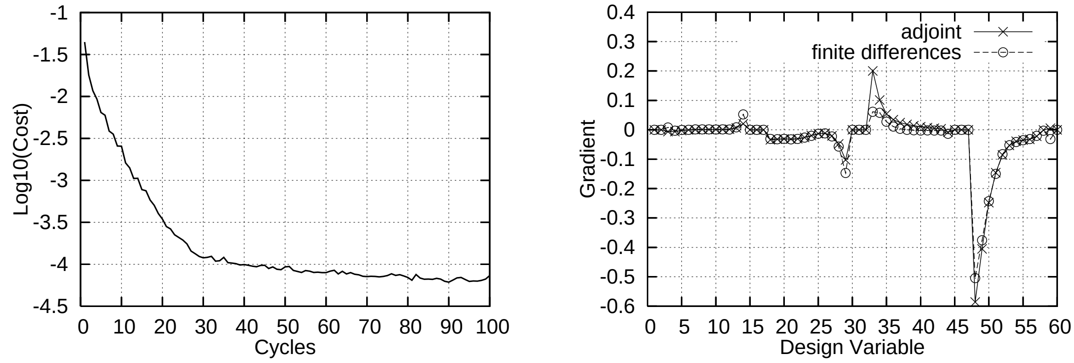


Figure 3: Case I: Convergence history (left) and objective function gradient components for the initial geometry computed using the preconditioned adjoint method and finite differences (right).

Case I is concerned with the inverse design of a NACA4415 airfoil at $M_\infty = 0.001$ and $a_\infty = 6°$. Fig. 2 shows the initial, reference and optimal airfoil contours and the corresponding pressure distributions. In this case, 30 control points are used. All but the leading and trailing edge control points are allowed to vary in both the chordwise and normal–to–chord directions, summing up to 56 design variables. In fig. 3, the convergence history of the optimization procedure is shown. Assuming that the solution of the flow and the adjoint equations are of almost equal CPU cost, the optimization costs $\sim$ 200 equivalent flow solutions. In fig. 3, the objective function gradient values computed with the preconditioned adjoint method are compared to those computed using finite differences. The first 30 data correspond to the chordwise (first 15) and normal–to–chord (16 to 30) control point coordinates of the pressure side "measured" from the trailing to the leading edge; the next 30 variables correspond to the control points parameterizing the suction side (same sequence). The need for preconditioning both the flow and adjoint equations becomes clear in fig. 4, where the speed–up of both the direct and adjoint equations is shown.

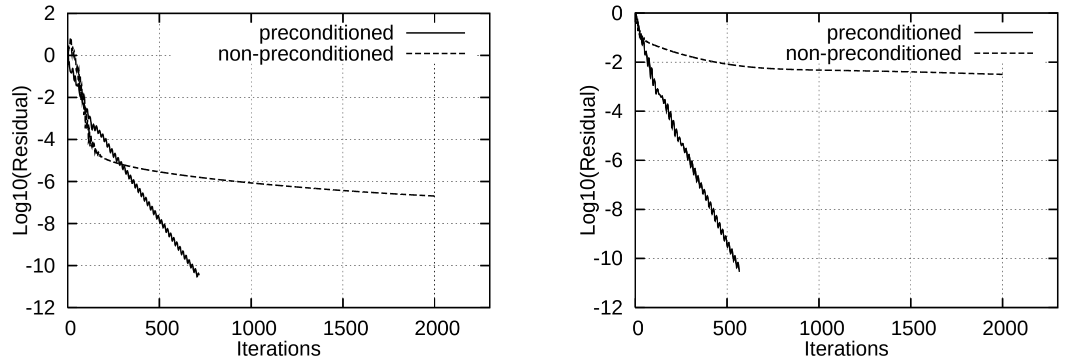
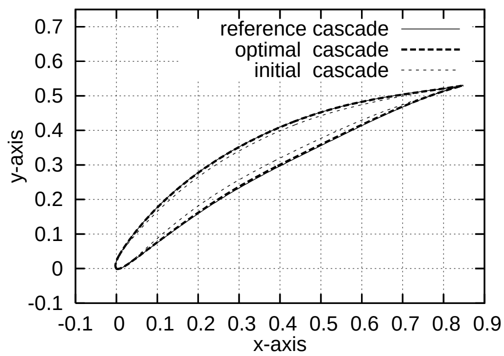
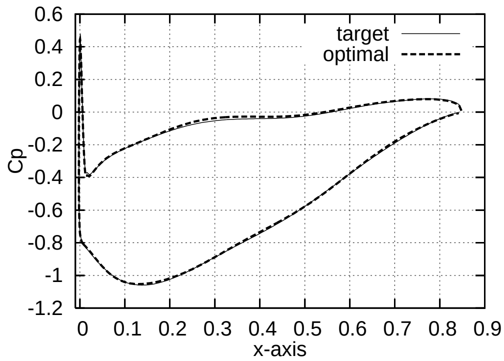


Figure 4: Case I: Convergence rates of the flow (left) and the adjoint (right) equations on a selected airfoil with and without preconditioning.

Case II is concerned with the inverse design of a compressor cascade in turbulent flow ($M_{2,is}$=0.1, a_1=50° and Re=8.10^5). The Spalart–Allmaras model with wall–functions[8] is used. A grid with triangular elements, generated using the advancing front technique and superimposed to structured–like layers of triangles stacked around the airfoil, are used. The optimization results, fig. 5, are satisfactory.





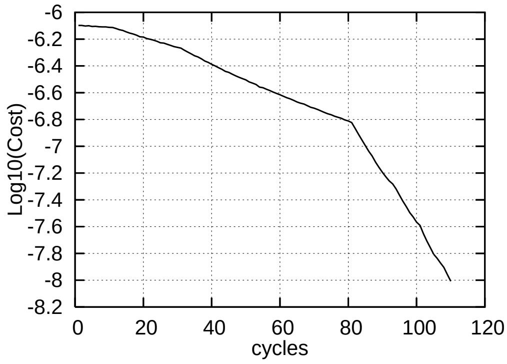


Figure 5: Case II: Initial, reference and optimal airfoil (top–left), corresponding pressure distributions (top–right) and convergence history (bottom).

5 CONCLUSIONS

A preconditioned continuous adjoint formulation for the inverse design of isolated and cascade airfoils, valid for both high and low Mach number flows was presented. The proposed formulation allows the inverse design optimization of aerodynamic shapes at very low Mach numbers with reasonable CPU cost, compared to the conventional adjoint approach (without preconditioning).

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